Sparsity Estimation in Image Compressive Sensing

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***Abstract*— Compressive sensing is an emerging technology which can recover a K*-sparse* signal vector from $M = O(Klog(K/N))$ measurements. However, it is a challenge to know exactly how many measurements an image requires to achieve an acceptable recovered visual quality. In this paper, we study the relationship between the image's complexity and its sparsity. We propose a mathematical model to estimate the number of needed measurements by using the image's texture, the edge density and the target reconstruction quality. There exists a linear function between them. The experimental results with a large number of photo pictures show that, quite most reconstructed images using our pre-calculated number of measurements have good enough quality, which confirms our proposed image-complexity-based model well.**

I. Introduction

The conventional approach of reconstructing signals from measured data follows the well-known Shannon sampling theorem, which states that the sampling rate must be twice the highest frequency. Similarly, the fundamental theorem of linear algebra suggests that the number of collected measurements of a discrete finite-dimensional signal should be at least as large as its dimension in order to ensure reconstruction. However, the novel theory of compressive sensing (CS) provides an alternative to Shannon/Nyquist sampling when the signal under acquisition is known to be sparse or compressible [1]–[3]. It states that if a signal is sparse, then under certain conditions it can be reconstructed exactly from a small set of non-adaptive, linear measurements using tractable optimization algorithms.

In CS, instead of taking N periodic signal samples, we measure $M \ll N$ inner products with measurement vectors. In matrix notation, the measurements $y = \Phi x$, where the rows of the $M \times N$ matrix Φ contain the measurement vectors. While the matrix is rank deficient, it loses the information in general. In their works [4], Candes, Romberg and Tao prove that if the matrix satisfies the *restricted isometry property* (RIP), which is initially called the *uniform uncertainty principle*, it can preserve the information in sparse and compressible signals. A large class of random matrices have the RIP with overwhelming probability, such as Gaussian, Bernoulli, Rademacher(± 1), partial random Fourier matrices, etc. To recover the signal from the compressive measurements y, we search for the sparsest coefficient vector θ that agrees with the measurements. Today's state-of-the-art CS systems can robustly recover K*-sparse* and compressible signals from just $M = O(Klog(K/N))$ noisy measurements using polynomial-time sparsity-seeking optimization solvers or greedy algorithms. Several introductory texts about compressive sensing, as well as a lot of reference materials can be found on [5].

In recent years, there have been growing interests in applying the results from the field of CS to imaging applications, an area known as compressive imaging. It is proved that CS is very effective in imaging [6], [7]. However, for compressible signals or images, the sparsity K is unknown, so the value of M is also undetermined. How many compressive measurements it requires to achieve an acceptable visual quality for a given image? To the best of our knowledge, this problem is substantially unexplored.

The sparsity is obviously relevant to the complexity of image content. There are a wide variety of definitions for image complexity. For example, in [8], the image complexity is related to the number of the objects and segments within it. Some works have related the image complexity to the entropy of image intensity [9]. In [10], the complexity has been considered as a subjective characteristic represented by a fuzzy interpretation of edges in an image. These definitions show that there are different approaches to represent the image complexity. Since each definition, based on either subjective or objective characteristics of the input image, uses a distinct measurement or calculation algorithm, therefore, there is not any agreement on the image complexity definitions.

In this paper, we first study how to represent image complexity, and propose a complexity metric using image texture and edge density. We propose a mathematical model based on image complexity to estimate the number for compressive measurements. With the training image set, we model a sparsity function with image complexity. We also explore the relationship between the measurement number and the image quality. It shows that it is nearly linear in a specific range, such as 26dB - 38dB. We verify the effectiveness of the model with a large number of natural images.The experimental results confirm our proposed complexity-based model well.

In summary, this paper makes three important contributions. Firstly, we give a method to represent the image complexity by image texture and edge density. Secondly, we propose a sparsity estimation algorithm based on image complexity. Last, we model the relationship between the number of the measurements and the target recovered quality.

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The remainder of the paper is organized as follows. In Section II, we introduce the background of compressive sensing. In Section III we firstly describe our image complexity metric by image texture and edginess, and present our proposed mathematical model based on the image complexity. Section IV presents the experiments and results. Finally conclusions are provided in Section V.

II. Background of Compressive Sensing

Consider a signal $x \in \Re^N$, which is K-sparse in an orthonormal basis Ψ with size $N \times N$; that is, $\theta \in \Re^N$ defined as $\theta = \Psi^T x$, has at most K nonzero components. Compressive sensing [1]–[4] deal with the recovery of x from undersampled linear measurements of the form:

$$y = \Phi x = \Phi\Psi\theta = A\theta, \tag{1}$$

where y is a $M \times 1$ vector, $\Phi \in \Re^{M\times N}$ is the measurement matrix that is incoherent with Ψ, and $A = \Phi\Psi$. More specifically, the M measurements in y are random linear combinations of the entries of θ, which can be viewed as the compressed and encrypted version of x. For $M < N$, estimating x from the measurements y is an ill-conditioned problem. Exploiting the sparsity of θ, CS states that the signal x can be recovered exactly from

$$M = O(Klog(N/K)) \tag{2}$$

measurements provided the matrix A satisfies the so-called *restricted isometry property* (RIP). Many random matrices whose entries are independent and identically distributed (i.i.d.) Gaussian, or more generally subgaussian have the RIP property. It has been shown that we can recover θ (or equivalently, x) exactly by solving the following l_0-norm minimization problem:

$$min\|\theta\|_0 \quad s.t.y = \Phi x = A\theta. \tag{3}$$

Unfortunately, it is a combinatorial, NP-hard problem; furthermore, the recovery is not stable in the presence of noise [3]. Stable recovery algorithms actually rely on the RIP. They can be grouped into two camps. The first approach convexifies the l_0-norm minimization (3) to the l_1-norm minimization

$$min\|\theta\|_1 \quad s.t.y = \Phi x = A\theta. \tag{4}$$

It corresponds to a linear program that can be solved in polynomial time. Many algorithms have been proposed to solve the convex optimization problem, including interior-point methods, projected gradient methods, and iterative thresholding.

The second approach finds the sparsest x agreeing with the measurements y through an iterative, greedy search. Algorithms such as matching pursuit, orthogonal matching pursuit, StOMP [12], CoSaMP [13], and Subspace Pursuit all build up an approximation one step at a time by making locally optimal choices at each step.

III. Image Complexity Based Compressive Sensing

For K-sparse signal, we can recover it with $M = O(Klog(N/K))$ measurements via sparsity-seeking optimization or greedy algorithms. The image, however, is not a sparse signal, but a compressible signal. The sparsity K of the image is unknown and non-obvious. So it is difficult to determine how many measurements a given image requires to achieve an acceptable visual quality. Moreover, the adaptivity is crucial to capture the regularity of complex natural images.

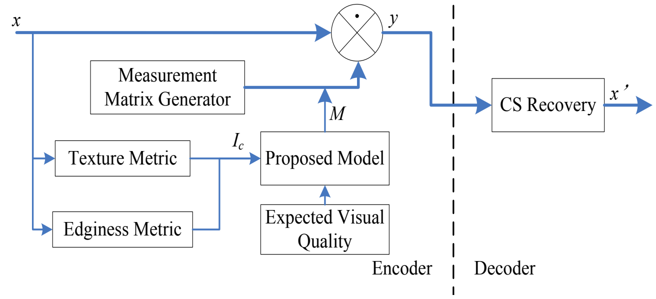

Fig. 1. The flow chart of proposed method.

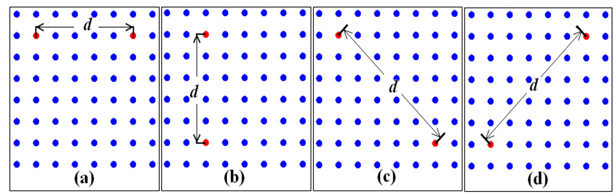

Fig. 2. Four different directions for distance d. ($d = 5$)

In this paper, we propose a framework to estimate the required number M of compressive measurements based on the image complexity. Here we concern the overall description of the image complexity so as to have a global grasp of the image data, but not other detail messages, such as the number of objects and segments within the image. Although there is not a unique method for the image complexity calculation, there is a global agreement in classifying images by complexity. Among all image features, the texture and the edginess are the two most important ones for the image visual complexity. Fig. 1 is the flow chart of our proposed method. In the next section, we firstly introduce our complexity measuring approach, including texture metric and edginess metric, and then propose our model to estimate the number of measurements of compressive sensing for a target reconstruction quality.

A. Texture Metric

To date, many measuring methods about the texture have been developed. Gray level co-occurrence matrix (GLCM) [14] is one of the most known texture analysis approaches. It estimates the image properties related to the second-order statistics. Each entry (i, j) in GLCM corresponds to the number of occurrences of the pair of gray levels i and j which are a distance d apart in the original image. The probability of gray level i to j is defined as $p_d(i, j)$. In general, there are 4 different directions for distance d, shown in Fig. 2. In order to estimate the similarity between different gray level co-occurrence matrices, Haralick [14] proposed 14 statistical features extracted from them. Among these features, the entropy measures the disorder of the image. Its mathematical equation is shown as:

$$Entropy = -\sum_i \sum_j p_d(i,j) log p_d(i,j). \tag{5}$$

The entropy achieves the largest value when all elements in GLCM matrix are equal, which implies it is a completely random image. When the image is texturally uniform, only few GLCM elements are large values, others are zero, which implies that the entropy is very small. The entropy gives us the average information or uncertainty of a random variable, which just corresponds to the image complexity.

In our experiments, we firstly divide the whole image into several regions with size 16×16, and calculate each one's entropy of GLCM. At last, the average entropy is defined as the image texture complexity, as in:

$$\begin{aligned} \overline{E}_{tex} &= \frac{1}{N_R} \sum_{k=1}^{N_R} Entropy_k \\ &= -\frac{1}{N_R} \sum_k \sum_i \sum_j p_d(i,j,k) log p_d(i,j,k), \end{aligned} \tag{6}$$

where N_R is the number of regions of the image, $p_d(i,j,k)$ is the probability of gray level i to j in region k. In our work, the distance $d = 5$ in Fig. 2 (a).

B. Edginess Metric

Excepted for the texture, the edginess is also a very important component for image visual complexity. An image which contains more prominent edges looks clearly more complex. In this paper we use the edge ratio as edginess metric, defined as in:

$$R_{edge} = \frac{N_{edge\_pixel}}{N_{total\_pixel}}, \tag{7}$$

where N_{edge_pixel} is the total number of edge pixels and N_{total_pixel} is the total number of pixels in the image. We make use of Prewitt edge detection method with threshold 0.04. The Prewitt operator calculates the gradient of the image intensity at each point, giving the direction of the largest possible increase from light to dark and the rate of change in that direction. Its result therefore shows how "abruptly" or "smoothly" the image changes at that point. It is exactly corresponding to the image complexity or sparsity.

C. Compressive Sensing Based on Image Complexity

Next, we introduce our proposed mathematical model, which describes the relationship between the image complexity and the number of needed measurements in compressive sensing for a certain expectant quality. From the above, we measure the image complexity I_c with the sum of the texture and the edginess of the image, as in

$$I_c = \overline{E}_{tex} + R_{edge}, \tag{8}$$

where $\overline{E}_{tex}$ is calculated with (6), R_{edge} is carried out with (7), and they are normalized in the range $(0, 1)$, respectively. Furthermore, we do experiments to recover the

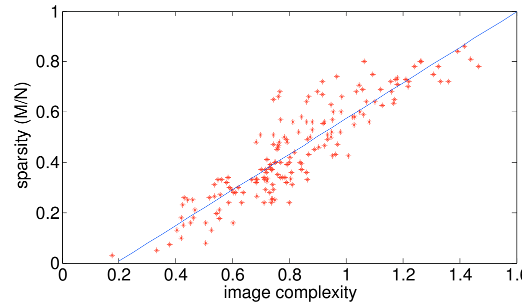

Fig. 3. The sparsity *vs* the complexity for training images. The line is the fitted result with least-square approximation.

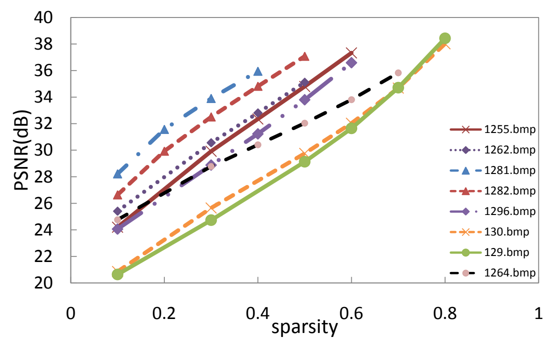

Fig. 4. The construction quality of some training images with different number of measurements.

image with a varying number of compressive measurements using Romberg's recovery algorithm in [6]. The reconstruction performance is measured by PSNR (dB). From the experiment results, we estimate the required M value for PSNR=32dB of the reconstructed image. Our training image set includes 100 images downloaded from USC SIPI image database. Fig. 3 presents the result about the image sparsity vs the complexity for the training images. And then we fit the experimental data with least-squares approximation to a linear function, as the line shown in Fig. 3. The fitted result is formulated as:

$$\widetilde{M} = f(I_c) = \alpha \cdot I_c + \beta, \tag{9}$$

where $\widetilde{M}$ is the estimated sparsity, that is the ratio of the measurements to the total number of pixels in the image, and α and β are fitting coefficients, I_c is the image complexity using (8). In our experiments $\alpha = 0.70, \beta = -0.24$. In Section IV we verify the model with a large number of test images.

D. Target Reconstruction Quality

For the image, the more measurements in compressive sensing, the better reconstruction quality. We observe that the recovery quality is almost linear to the number of compressive measurements in a general range, and the lines' slopes are nearly the same. The construction quality of some training images with different number of compressive measurements are shown in Fig. 4. So we can revise the model (9) to :

$$\widetilde{M}_T = \lambda \cdot (T - 32) + \widetilde{M}, \tag{10}$$

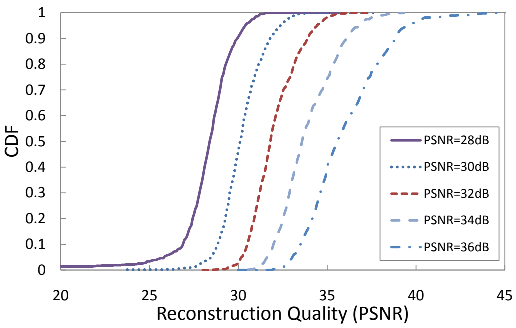

Fig. 5. CDF of recovery PSNR using pre-calculated sparsity for different target reconstruction quality.

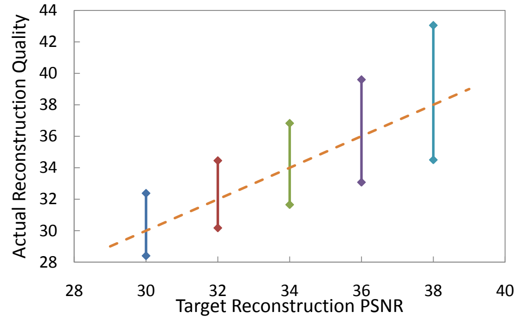

Fig. 6. The actual reconstruction quality *vs* target PSNR.

where T is the target reconstructioin quality, λ is the quality parameter, which is the average slope of training image set, $\widetilde{M}$ is the calculated sparsity using (9) for PSNR = 32dB. In our experiments $\lambda = 0.04$. In next section we verify the updated model.

IV. Experiments and Results

The experiments are conducted as follows. Firstly, the test image set contains 5000 images in good quality. 3000 of them are downloaded from the USDA NRCS Photo Gallery. Another 2000 images are collected from several types of digital cameras. All images are resampled to make all the images in the size 256×256 and converted into gray-scale.

At first, we validate the sparsity model based on image complexity. Given a required PSNR, the number of compressive measurements is calculated using the proposed model. Each image is sampled with the number and then is recovered using Romberg's algorithm[6]. The quality of recovered image is measured by PSNR. Fig. 5 illustrates the cumulative distribution function (CDF) of reconstructed quality for all images. It demonstrates that, for a given target quality, most of images can be recovered with good enough visual quality.

Fig. 6 presents the relationship between the actual recovery quality and the target PSNR. It plots the 95% confidence interval of the reconstruction quality for all images. It can be seen that there exists a linear function between them. We also notice that the target PSNR curve locates well in the confidence interval. It shows that our model is accurate to estimate the image sparsity under a given target quality. In short, the above results demonstrate that our proposed sparsity estimation model is workable and suitable for compressive image sensing.

V. Conclusions

Compressive sensing is a new research topic in signal processing which has promoted wide research interests in various fields. In this paper, we propose a image-complexity-based sparsity model to estimate the number of compressive measurements. Among lots of image features, we use the image texture and the edginess as the metric of its complexity. The experimental results with a large number of real-world images show that, most of reconstructed images have achieved the target visual quality. In future, we will further check our model on more used resolutions (e.g. 720p, 1080p), and will exploit more relationships between the visual features and the image sparsity.

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