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# Dolha - an efficient and exact data structure for streaming graphs

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**Abstract**

A streaming graph is a graph formed by a sequence of incoming edges with time stamps. Unlike the static graphs, the streaming graph is highly dynamic and time-related. Streaming graphs in the real world, which are of the high volume and velocity, can be challenging to the classic graph data structures: data of internet traffic, social network communication, and financial transections, etc. The traditional graph storage models like the adjacency matrix and the adjacency list are no longer sufficient for the large amount data and high frequency updates. And most the streaming graph structures are only supports the specific graph algorithms. Here a new data structure is presented to meet the challenge: a double orthogonal list in hash table (Dolha) as a high speed and high memory efficiency graph structure. Dolha has constant time cost for single edge processing, and near-linear space cost. Moreover, time cost for neighborhood queries in Dolha is linear, which enables it to support most algorithms of graphs without extra cost. A persistent structure based on Dolha is also presented, to handle the sliding window update and time related queries.

**Keywords** Streaming graph · Data structure · Efficient and exact · Graph algorithms

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## 1 Introduction

In the real world, billions of relations and communications are created every day. A large ISP needs to deal about $10^9$ packets of network traffic data per hour per router [16]; 100 million users are active on Twitter with around 500 million new tweets per day [31]; In worldwide, the total number of sent/received emails are more than 200 billion per day [11]. Yet these newly created relations and information fade away just as quickly as tides. Mining knowledge from these highly dynamic data is as difficult as capturing a wave in the sea. A graph data structure of high efficiency for both the memory capacity and speed is thus proposed, to deal with challenges in the storage of such enormous amount of data, and seizing and salvaging data every nanosecond from the stream.

There have been several prior arts in streaming graph summarization like TCM [30] and and specific queries like TRIÈST [9]. However, there are still situations that are not covered in these existing work . To illustrate, some motivation examples are briefed as follows:

The network traffic is a typical kind of streaming graphs. Each IP address indicates one vertex and the communication between two IPs indicates an edge. With the data packets being sent and received between the IPs, the graph changes rapidly. The existing graph summarization techniques (such as TCM [30]) supports the vertex query and edge query in $O(1)$ time cost. However, they cannot suffice for the queries that are more structure-aware, such as queries about "the receivers of given IP", "the 2-hop neighbors of a specific IP" and "the counts of IPs that a certain IP reaches. In some applications, an *exact* data structure is desirable for streaming graphs rather than probabilistic data structure. In a social network graph, the system needs to face even more complicated queires such as triangle counting, subgraph matching, social influence [23] and trustworthy [21] algorithms. But existing solutions are designed specifically for triangle counting [9] and so are some continuous subgraph matching systems [18], circle detecting systems [27] and social trust path [19, 20, 22] over streaming graphs. To run various kinds of graph analysis, multiple streaming systems must be maintained with high cost on both space and time. An optimized solution is thus needed as a uniformed system that could support most current analysis algorithms on streaming graphs.

Moreover, an edge in streaming graph is received with a time-stamp. Based on these time stamps, **historical information** or **time constrains** are figured out and commonly called in multiple applications and scenarios, yet few systems support these time-related graph queries. For example, if there are a few suspicious financial transections made on Tuesday between 10am and 4pm out through the bank, the needs to run a pattern match on the transfers within such a given time frame. Another example is the credit card fraud detection.If we have the account IDs of the major parties involved in a credit card fraud, a set of query graphs is constructed by considering these IDs as vertex and the transactions as edges. We need to locate the occurrence time when these query graphs appear in the streaming transaction graph, then we check inward and outward neighbors of these suspicious accounts near that time and find other criminal group members. In these cases, the streaming graph system should not only support *last snapshot*-based queries, but also the *time-related* queries for figuring out historical information to be further called in the match.

Motivated by above use cases, an efficient streaming graph structure should suffice the requirements below:

– To enable efficient graph computing, the space cost of the data structure should be small enough to fit into main memory to ensure efficient graph computing;

- For the enormous amount of data and the high-frequency updating, the data structure must have $O(1)$ time cost to handle one incoming edge processing;
- The data structure should support many kinds of graph algorithms rather than designed for one specific graph algorithm;
- The data structure should also support time-related queries for historical information.

There are two kinds of graph data structures in mainstream literature : general streaming graph data structure and a data structure designed for some specific graph algorithms. General streaming graph structure is designed to preserve the whole structure of streaming graphs, thus, it supports most of graph algorithms such as BFS, DFS, reachability query and subgraph matching by using neighbor search primitives. Most of its variations are based on hash map associated with some classical graph data structures, such as adjacency matrix and adjacency list. For example, GraphStream Project [26] is based on adjacency list associated with hash map. The basic idea of this structure is to map the vertex IDs into a hash table. Each cell of vertex hash table stores the vertex ID and the incoming/outgoing links. TCM [30] and gMatrix [17] propose to combine hash map with adjacency matrix. Different from [26], TCM and gMatrix are approximate data structures that inherit query errors due to hash conflicts. There are also some other streaming graph data structures that support a *specific* graph algorithm solely, such as HyperANF [3] for t-hop neighbor and distance query, the Single-Sink DAG [13] for pattern matching and TRIEST [9] for triangle counting.

Table 1 lists the space cost of different general streaming graph data structure, together with the time complexity to handle edge insertion and edge/1-hop queries. GraphStream's $O(d)$ edge insertion time is dependent to the maximum vertex degreewhich can be very large in free-scale network data. Thus, GraphStream is not suitable for high speed streaming graph. TCM and gMatrix have the square space cost that prevents them to be used in large graphs. The approach (called Dolha) proposed in this paper improves the performance by fulfilling all requirements for streaming graphs.Dolha combines the orthogonal list with hash techniques. The orthogonal list builds two single linked lists of the outgoing and incoming edges for each vertex, and stores the first items of two lists in vertex cell. On the other hand, the hash table is commonly used for streaming data structure to achieve amortized $O(1)$ time look up, such as bloom filter [4] and count-min [8]. The combination of orthogonal list and hash table is a promising option to achieve our goal. Based on this idea, we present a new exact streaming graph structure: *d*ouble *o*rthogonal *l*ist in *ha*sh table (Dolha).

**Our contributions** Table 1 shows the comparison among the three general streaming graph structures. In this paper:

1. In Section 2, we introduce the two classes of streaming graph structures.

**Table 1** General streaming graph structures

| | Adjacency List | Adjacency Matrix | Orthogonal List |
|---|---|---|---|
| +Hash | GraphStream [26] | TCM [30] | Dolha |
| Space Cost | $O(\|E\| \log \|V\|)$ | $O(\|V\|^2)$ | $O(\|E\| \log \|E\|)$ |
| Time Cost per Edge | $O(\log d)$ | $O(1)$ | $O(1)$ |
| Edge Query | $O(\log d)$ | $O(1)$ | $O(1)$ |
| 1-hop Neighbor Query | $O(d)$ | $O(\|V\|)$ | $O(d)$ |

2. In Section 3, we define the streaming graph models and related algorithms.
3. In Section 4, we design an effective data structure (Dolha Snapshot) for snapshot model of streaming graphs with $O(|E|\log|E|)$ space cost and $O(1)$ time cost for a single edge operation.
4. In Section 5, we design a variant of Dolha (Dolha persistent) that supports sliding window and time related queries in linear time cost.
5. In Section 6, we design the graph algorithms both on Dolha Snapshot and Dolha Persistent.
6. In Section 7, we show experiments on both real and synthetic datasets have confirmed the capacity of Dolha and the improvements over the state-of-the-art.

## 2 Related work

Among the existing studies, the data structures can be categorized into two classes: general streaming graph structures and streaming graph algorithms structures.

### 2.1 General streaming graph structures

General streaming graph structures are designed to preserve the data of graph stream and maintain the graph connection information at the same time. General streaming graph structures support most of graph algorithms like BFS, DFS, reachability query and subgraph matching by using neighbor search primitives. Most structures of this kind are based on hash map associated with basic graph data structure, such as adjacency matrix and adjacency list. There are two kinds of streaming graph structures in general: exact structure and approximation structure (Figure. 1).

**Exact structures** Graph Stream Project [26] is an exact graph stream processing system implanted by Java. Graph Stream Project is based on adjacency list associated with hash map and it supports most of the mainstream graph algorithms. The structure is to map the vertex IDs into a hash table. Each cell of vertex hash table stores the vertex ID and its incoming / outgoing links.

Adjacency list needs $O(|E|\log|V|)$ space and $O(|V|+|E|)$ time for traversal. However, to locate an edge, we need to go through the neighbor lists pf both in and out vertices which indicates $O(|E|)$ time cost in some extreme situations. Even the neighbor lists are gathered into a sorted list, it still costs $O(\log d)$ time ($d$ is the average degree of vertices) for each edge look up.

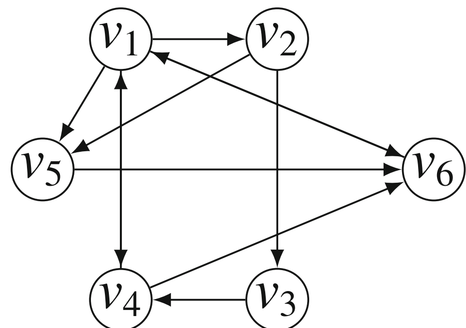



**Figure 1** Graph example

| Vertex Index | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Vertex ID | $v_2$ | | $v_6$ | | $v_1$ | | $v_4$ | | $v_3$ | | $v_5$ | |
| | out | in | out | in | out | in | out | in | out | in | out | in |
| | 4 | 2 | 2 | 2 | 0 | 1 | 1 | 2 | 3 | 0 | 1 | 0 |
| | 5 | | | 3 | 1 | 3 | 2 | 4 | | | | 2 |
| | | | | 5 | 3 | | | | | | | |
| | | | | | 5 | | | | | | | |

**Figure 2** The adjacency list in hash table for the graph example

Figure 2 shows the adjacency list in hash table for the graph example (1). The hash function $H(*)$ is used to map the 6 vertices into 6 cells vertex hash table, and each cell has 2 sorted list to store the outgoing and incoming neighbors of the vertex. i.e., $H(v_2) = 0$ and cell 0 stores the vertex ID $v_2$, the outgoing list $\{4 = H(v_3), 5 = H(v_5)\}$ and incoming list $\{2 = H(v_1)\}$. The adjacency list stores the exact information of the graph stream but cost $O(d)$ for each edge insertion.

**Approximation structures** the adjacency matrix in hash table is the other major kind of structure as the solution for streaming graph. We could hash the vertices into a hash table, and then use a pair of vertices indexes as coordinates to construct an adjacency matrix. Vertex query in hash table is $O(1)$ time cost and so is edge look-up in the matrix. Adjacency matrix in the hash table is efficient timewise, but $O(|V|^2)$ space cost is a drawback. In the real world, graphs are usually sparse and we could not afford to spend 2.5 quadrillion on a 50 million vertices graph. There is a compromise formula that we compress the vertices into $O(\sqrt{|E|})$ size or even smaller hash table, to reduce the space cost up to $O(|E|)$. Due to the high compress ratio, it's only suite for a graph summarization system, like TCM [30], gMatrix [17].

Figure 3 shows the adjacency matrix in hash table for the graph example (1). We use hash function $H(*)$ to map the 6 vertices into 3 cells hash table and use the table index to build a $3 \times 3$ matrix. In the 9 cells of the matrix, we store the weights of 11 edges. i.e., $H(v_1) = 1$ and $H(v_2) = 0$, the matrix table cell $(1, 0)$ indicates the edge $\overrightarrow{v_1 v_2}$. However, the cell $(1, 0)$ also indicates the edge $\overrightarrow{v_1, v_6}$ and $\overrightarrow{v_5, v_6}$ since the hash collision. If we do outgoing neighbor

**Figure 3** The adjacency matrix in hash table for the graph example

| Vertex Index | 0 | 1 | 2 |
|---|---|---|---|
| Vertex Label | $v_2, v_6$ | $v_5, v_1$ | $v_3, v_4$ |

| | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 3 | 1 |
| 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 1 |

query for $v_2$, the result is $\{v_5, v_1, v_2, v_3\}$ and the correct answer is $\{v_5, v_3\}$. In this case, if we want the exact result, the matrix size is $6 \times 6$ which is much larger than the edge size 11.

### 2.2 Specific streaming graph structures

Unlike the general structure, there are some data structures designed for specific algorithms on streaming graph. For example, HyperANF [3] is an approximation system for t-hop neighbor and distance query; the Single-Sink DAG [13] is for pattern matching on large dynamic graph; TRIÈST [9] is sampling system for triangle counting in streaming graph; SpotLight [12] is a randomized sketching-based system that detects the sudden appearance of the high dense subgraphs on the streaming graph; both [7] and [25] are focusing on the graph classification model of streaming graph; and there are some connectivity and spanners structures showed in Graph stream survey [24]. These structures support the algorithms for the designed purpose, and are not feasible for other graph queries.

Time constrained continuous subgraph search over streaming graphs [18] is the latest research body of work that considers the time as a query parameter. This paper proposed an a kind of query that requires not only the structure matching but also the time order matching. Figure 4 shows an example of time constrained subgraph query. In this query, each edge of query graph has a time-stamp constrain $\epsilon$. A matched subgraph means the subgraph is an isomorphism of query graph and the time-stamps are following the given order.

## 3 Problem definition

**Definition 1** (**Streaming Graph**) A streaming graph $\mathcal{G}$ is a directed graph formed by a continuous and time-evolving sequence of edges $\{\sigma_1, \sigma_2, ...\sigma_x\}$. Each edge $\sigma_i$ from vertex $u$ to $v$ is arriving at time $t_i$ with weight $w_i$, denoted as $\sigma_i(\overrightarrow{uv}, t_i, w_i)$, $i = 1, ..., x$.

Generally, there are two models of streaming graphs in the literature. One is only to care the latest snapshot structure, where the latest snapshot is the superposition of all coming edges to the latest time point. The other model records the historical information of the streaming graphs. The two models are formally defined in Definitions 2 and 4, respectively. In this paper, we propose a uniform data structure (called *Dolha*) to support both of them.

**Definition 2** (**Snapshot & Latest Snapshot Structure**) An edge $\overrightarrow{uv}$ may appear in $\mathcal{G}$ multiple times with different weights at different time stamps. Each occurrence of $\overrightarrow{uv}$ is denoted as $\sigma^j(\overrightarrow{uv}, t^j, w^j)$, $j = 1, .., n$. The total weight of edge $\overrightarrow{uv}$ at snapshot $t$ is the weight sum of all occurrences before (and including) time point $t$, denoted as

$$W^t(\overrightarrow{uv}) = \sum_{t^j \leq t} w^j.$$

where $\sigma(\overrightarrow{uv}, t^j, w^j)$ appears in streaming graph $\mathcal{G}$.

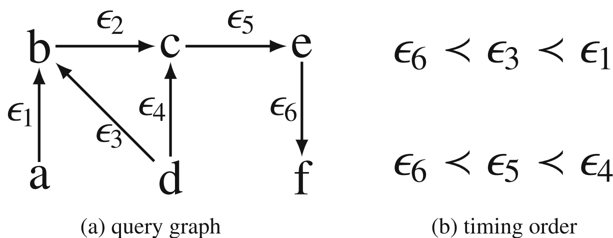



**Figure 4** Running example query $Q$ (Taken from [18])

For a streaming graph $\mathcal{G}$, the corresponding snapshot at time point $t$ (denoted as $\mathcal{G}_t$) is a set of edges that has positive total weight at time $t$:

$$\mathcal{G}_t = \{(\overrightarrow{uv}) \in \mathcal{G} \mid W^t(\overrightarrow{uv}) > 0\}.$$

When $t$ is the current time point, $\mathcal{G}_t$ denotes the *the latest snapshot structure* of $\mathcal{G}$.

An example of streaming graph $\mathcal{G}$ is shown in Figure 5. Figure 6 shows the snapshots of $\mathcal{G}$ from $t_7$ to $t_{10}$. In Figure 6c, total edge weight $\overrightarrow{v_1v_2}$ is updated from $W^1(\overrightarrow{v_1v_2}) = 1$ (at time $t_1$) to $W^7(\overrightarrow{v_1v_2}) = 2$ (at time $t_7$). In Figure 6d, edge $\overrightarrow{v_1v_4}$ receives a negative weight update. Since the weight of $\overrightarrow{v_1v_4}$ is 0 after update, it means that it is deleted from the snapshot $\mathcal{G}_8$ at time $t_8$. In Figure 6e, the deletion of edge $\overrightarrow{v_1v_2}$ causes the deletion of vertex $v_1$ in $\mathcal{G}_9$ and $v_1$ is added into $\mathcal{G}_{10}$ again because the new edge $\overrightarrow{v_1v_2}$ incoming at $t_{10}$.

**Applications** The snapshot structure stores the last updated information of the streaming graph. On the snapshot structure, we could perform **BFS**, **DFS**, **reachability** query, **triangle finding** et al. to acquire the latest struction information of the streaming graph and solve the problems like vertices and edges queires, social network algorithms.

In some **applications**, we need to record the historical information of streaming graphs, such as financial transaction and fraud detection example in Section 1. Thus, we also consider the sliding window-based model (Figure 7).

**Definition 3** (**Sliding Window**) Let $t_1$ be the starting time of a streaming graph $\mathcal{G}$ and $w$ be the window length. In every update, the window would slide $\theta$ and $\theta < w$. $D^i_{w,\theta}(\mathcal{G})$ contains all edges in the $i$-th sliding window, denoted as:

$$D^i_{w,\theta}(\mathcal{G}) = \{(\overrightarrow{uv}, t, w) |$$

$$(\overrightarrow{uv}, t, w) \in \mathcal{G}, t_0 + (i-1) \times \theta \le t \le t_0 + (i-1) \times \theta + w\}.$$

[14]

In Figure 7, the window size $w = 7$ and each step the window slides $\theta = 3$ edges. Figure 7 illustrates the first and the second sliding window, where the left-most three edges expired in the second window.

**Definition 4** (**Window Based Persistent Structure**) Given a streaming graph $\mathcal{G}$, the **Window Based Persistent Structure** ("persistent structure" for short) is a graph formed by all the unexpired edges in the current time window. Each edge is associated with the time stamps denoting the arriving times of the edge. An edge may have multiple time stamps due to the multiple occurrences.

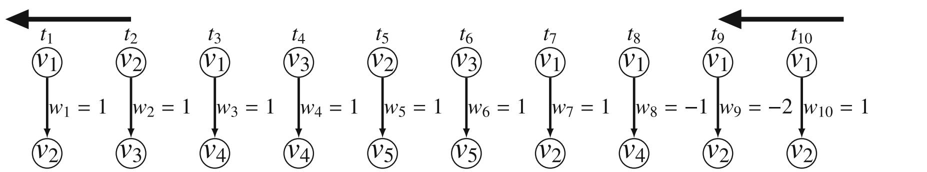



**Figure 5** Streaming graph $S$

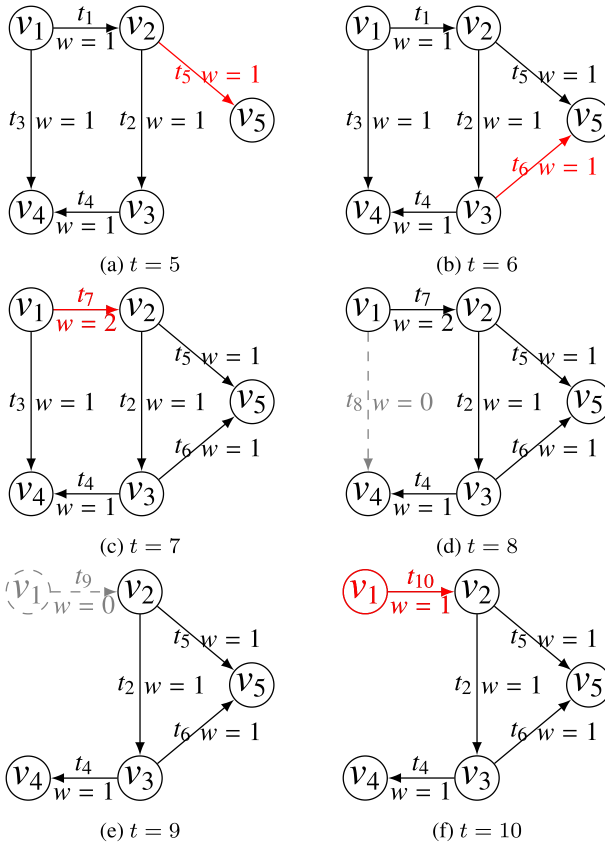



**Figure 6** Snapshot $\mathcal{G}_5$ to snapshot $\mathcal{G}_{10}$ of streaming graph $\mathcal{G}$

In a snapshot streaming graph structure, only the latest snapshot is recorded and the historical information is overwritten. For example, a snapshot structure only stores the snapshot $\mathcal{G}_{10}$ at last time point $t_{10}$ in Figure 6f. The update process of the streaming graph is overwritten.

Assume that the second time window (Window 1) is the current window. Figure 8 shows how the persistent structure stores the streaming graph. Edge $\overrightarrow{v_1 v_2}$ is associated with three time points ($t_7$, $t_9$ and $t_{10}$) that are all in the current time window. Although edge $\overrightarrow{v_1 v_2}$ also occurs at time $t_1$, it is expired in this time window. The gray edges denotes all expired edges, such as $\overrightarrow{v_1 v_4}$ and $\overrightarrow{v_2 v_3}$.

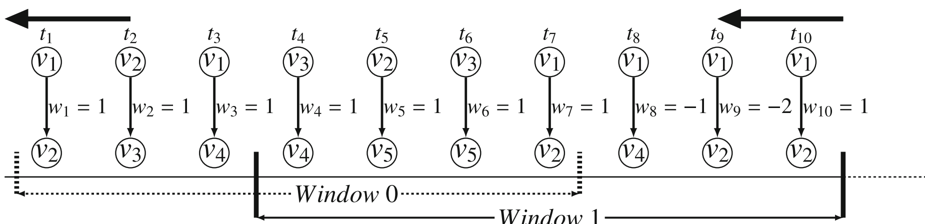



**Figure 7** Sliding window update on streaming graph

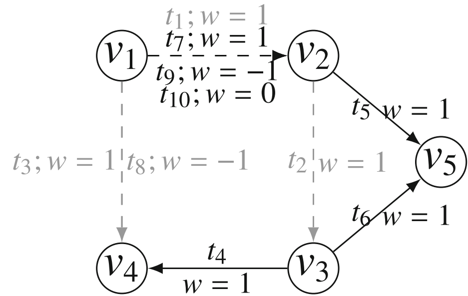



**Figure 8** Window based persistent structure

**Definition 5** (**Streaming graph query primitives**) We define 4 query primitives for streaming graph $\mathcal{G}$:

1. **Edge Query:** Given the a pair of vertices IDs $(u, v)$, return the weight or time stamp of the edge $\overrightarrow{uv}$. If the edge doesn't exist, return $\{null\}$.
2. **Vertex Query:** Given the a vertex IDs $u$, return the incoming or outgoing weight of $u$. If the vertex does not exist, return $\{null\}$.
3. **1-hop Successor Query:** Given the a vertex IDs $u$, return a set of vertices that $u$ could reach in 1-hop. If there is no such vertex, return $\{null\}$.
4. **1-hop Precursor Query:** Given the a vertex IDs $u$, return a set of vertices that could reach $u$ in 1-hop. If there is no such vertex, return $\{null\}$.

The query primitives are slightly different in two structures. If we query edge $\overrightarrow{v_1v_2}$ in snapshot structure at $\mathcal{G}_{10}$, the result is the last updated edge information : $(\overrightarrow{v_1v_2}, t_{10}, 1)$. If we query edge $\overrightarrow{v_1v_2}$ in persistent structure at $\mathcal{G}_{10}$ showing in Figure 8, the result is a list of unexpired edges: $(\overrightarrow{v_1v_2}, t_7, 0)$, $(\overrightarrow{v_1v_2}, t_9, -1)$, $(\overrightarrow{v_1v_2}, t_{10}, 1)$. The same difference applies to 1-hop successor query and precursor query. If we query the successor of $v_1$ at $t_{10}$, the snapshot structure will give the answer $v_2$. But the persistent structure will return a set of answers: $(v_2, t_7)$, $(v_2, t_9)$, $(v_2, t_{10})$.

Based on the persistent structure query primitives, we define a new type of queries on streaming graph named *time related query* that considers the time stamps as query parameters. In this paper, we adopt two kinds of time related queries: time constrained pattern query is to find the match subgraph in a given time period; structure constrained time query is to find the time periods that given subgraph appears in $\mathcal{G}$.

**Definition 6** (**Time Constrained Pattern Query**) A pattern graph is a triple $P = (V(P), E(P), L)$, where $V(P)$ is a set of vertices in $P$, $E(P)$ is a set of directed edges, $L$ is a function that assigns a label for each vertex in $V(P)$. Given a pattern graph $P$ and a time period $(t, t')$ and $t < t'$, $\mathcal{G}$ is a time constrained pattern match of $P$ if and only if there exists a bijective function $F$ from $V(P)$ to $V(g)$ such that the following conditions hold:

1. **Structure Constraint (Isomorphism)**
   - $\forall u \in V(P), L(u) = L(F(u))$.
   - $\overrightarrow{uv} \in E(P) \Leftrightarrow \overrightarrow{F(u)F(v)} \in E(g)$.
2. **Time Period Constraint**
   - $\forall \overrightarrow{uv} \in E(P), t \leq t_{\overrightarrow{uv}} \leq t'$.[18]

In this paper, the problem is to find all the time constrained pattern matches of given $P$ over $\mathcal{G}_{t'}$ which is the snapshot of $\mathcal{G}$ at time $t'$.

Figure 9 shows an example of time constrained pattern query. In Figure 9a, a pattern graph is given which queries all the 2-hop connected structures. The edges of pattern graph have a time constrain that only the edges with the time stamp between $(t_4, t_7)$ are considered as match candidates. Figure 9b is the snapshot $\mathcal{G}_7$ of $\mathcal{G}$ at time $t_7$. Edge $\overrightarrow{v_1v_4}$ and $\overrightarrow{v_2v_3}$ are discarded since the time stamps are out of time constrain. Edge set $\{(\overrightarrow{v_1v_2})(\overrightarrow{v_2v_5})\}$ is the only matching subgraph for the given pattern on $\mathcal{G}$.

**Definition 7** (**Structure Constrained Time Query**) A query graph $Q$ is a sequence of directed edges $\{q_1, q_2, ..., q_m\}$ and $T$ is a set of time pairs $\{(t_1, t_1'), ..., (t_n, t_n')\}$. Given a pattern graph $Q$, a structure constrained time match $T$ is that $Q$ is the subgraph of every snapshot of $\mathcal{G}$ during any time period $(t_i, t_i')$ in $T$.

$$\forall t, t_i \leq t \leq t_i', Q \in G_t.$$

Figure 10 gives an example of structure constrained by the time query edge set $\{\overrightarrow{v_1v_2}, \overrightarrow{v_2v_3}, \overrightarrow{v_3v_4}\}$. In this query, we have the topology of the query graph( Figure 10) and look for the time period(s) that the query graph existed. On $\mathcal{G}$, the query graph is the subgraph of every snapshot from $\mathcal{G}_4$ to $\mathcal{G}_8$ until deletion of $\overrightarrow{v_1v_2}$ on $\mathcal{G}_9$. In $\mathcal{G}_{10}$, the query graph is matching again since the new arriving $\overrightarrow{v_1v_2}$. The query result of Figure 10 is $\{(t_4, t_7), (t_{10}, t_{10})\}$.

**Applications** By using the window based persistent structure, we could maintain a time window based streaming graph and perform time related queries on it. In the example of Section 1, the financial transaction query could be solved by the Time Constrained Pattern Query. And the fraud detection could be solved by the Structure Constrained Time Query (Table 2).

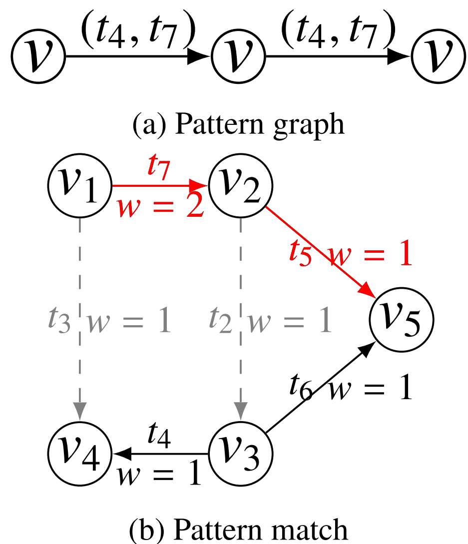
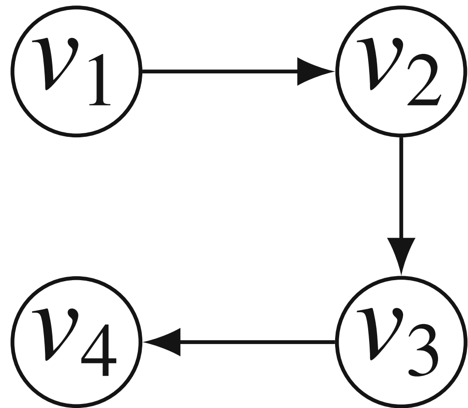



**Figure 9** Time constrained pattern query

Figure 10 Structure constrained time query




## 4 Dolha - double orthogonal list in hash table

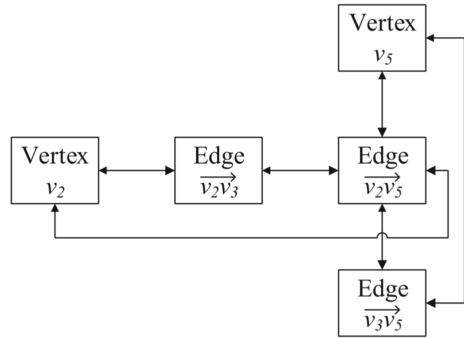
A data structure is proposed for processing the high speed streaming graph data, namely, the Double Orthogonal List in Hash Table (*Dolha* for short). Essentially, Dolha is the combination of double orthogonal linked list and hash tables. A *d*ouble *o*rthogonal *l*inked *l*ist (*Doll* for short) is a classical data structure to store a graph, in which each edge $\overrightarrow{uv}$ in graph $\mathcal{G}$ is both in the double linked list of all the outgoing edges from vertex $u$: $\{\overrightarrow{uv_A}, ...\overrightarrow{uv_\Omega}\}$ denotes as *outgoing Doll* and in the double linked list of all the incoming edges to vertex $v$: $\{\overrightarrow{u_A v}, ...\overrightarrow{u_\Omega v}\}$ denotes as *incoming Doll*. Vertex $u$ has two pointers: one is to the first item $v_A$ and the other is to last item $v_\Omega$ of outgoing Doll. Vertex $v$ has two pointers: one is to the first item $u_A$ and the other is to the last item $u_\Omega$ of incoming Doll.

For example, Figure 11 illustrates an example of Doll. The edge $\overrightarrow{v_2v_3}$ and $\overrightarrow{v_2v_5}$ are connected by the outgoing Doll of vertex $v_2$. The edge $\overrightarrow{v_2v_5}$ and $\overrightarrow{v_3v_5}$ are connected by the

Table 2 Notations

| Notation | Definition and description |
|---|---|
| $G_s$ / $G_t$ | Streaming graph / Snapshot at time point $t$ |
| $D_s$ / $D_p$ | Dolha snapshot / Dolha persisdent |
| $\overrightarrow{uv}$ | The directed edge from vertex $u$ to $v$ |
| Doll | Doulble orthogonal linked list |
| $O$ | Outgoing Doll |
| $I$ | Incoming Doll |
| $T$ | Time travel linked list |
| $w$ | Edge weight |
| $t$ | Edge time stamp |
| $H(*)$ | Hash value of $*$ |
| $V(*)$ | Vertex table index of $*$ |
| $E(*)$ | Edge table index of $*$ |
| $E_A^*()$ | First item's edge table index of link $*$ |
| $E_\Omega^*()$ | Last item's edge table index of link $*$ |
| $E_\Omega^*()$ | Last item's edge table index of link $*$ |
| $E_N^*()$ | Next item's edge table index of link $*$ |
| $E_P^*()$ | Previous item's edge table index of link $*$ |
| $*^{-/+}$ | Previous/next item of $*$ |

**Figure 11** Example of doll




incoming Doll of vertex $v_5$. Vertex $v_2$ has 2 pointers for the outgoing Doll: the first item pointer pointing to $\overrightarrow{v_2v_3}$ and the last item pointer pointing to $\overrightarrow{v_2v_5}$. Vertex $v_5$ has 2 pointers for the incoming Doll: the first item pointer pointing to $\overrightarrow{v_2v_5}$ and the last item pointer pointing to $\overrightarrow{v_3v_5}$.

## 4.1 Dolha snapshot data structure

Given a graph $\mathcal{G}$, the Dolha structure contains four key-value tables. First, we assume that each vertex $u$ (and edge $\overrightarrow{uv}$) is hashed to a hash value $H(u)$ (and $H(\overrightarrow{uv})$). For example, we use hash function $H(*)$ to map the vertices and edges:

- $H(v_1) = 1$, $H(v_2) = 2$, $H(v_3) = 0$, $H(v_4) = 1$, $H(v_5) = 3$
- $H(\overrightarrow{v_1v_2}) = 1$, $H(\overrightarrow{v_2v_3}) = 0$, $H(\overrightarrow{v_1v_4}) = 4$, $H(\overrightarrow{v_3v_4}) = 2$, $H(\overrightarrow{v_2v_5}) = 4$, $H(\overrightarrow{v_3v_5}) = 3$

**Vertex hash table** Dolha creates $m_v(m_v \geq |V|)$ size vertex hash table and the function $H(*)$ to map the vertex ID $u$ to vertex hash table index $H(u)$. Due to the hash collision, there would be a list of vertices with same hash table index. In each table cell, Dolha stores the vertex table index of the first vertex in the collision list.

Table 3 is an example of vertex hash table. We use $H(v_1) = 1$ as hash index to locate the vertex table index 0 and find the details of $v_1$ in vertex table cell 0. The vertex $v_4$ has the same hash value as $v_1$ which indicates a hash collision. We use hash value 1 to find the first vertex $v_1$ on the collision list then to find the next item $v_4$'s vertex table index 3 in $v_1$'s vertex table cell.

**Vertex table *V*** Dolha creates $m_v(m_v \geq |V|)$ size vertex table. A empty cell variable denoted as $\phi_V$. Initially, $\phi_V = 0$ . We use the vertex table index for new coming vertex $u$ as $V(u)$. Let $V(u) = \phi_V$ and increase $\phi_V$ by 1. In each vertex table cell, Dolha stores the vertex ID, the outgoing weight sum $w_O(u)$, the incoming weight sum $w_I(u)$ and the head and tail edge table index for outgoing Doll, the head and tail edge table index for incoming Doll and and the vertex table index of the next vertex on collision list.

Table 4 shows the vertex table of $\mathcal{G}_5$ in Figure 6. Out/In $w$ indicates the outgoing and incoming weights of the vertex. $O$ is the edge table index of first and last items of outgoing Doll; $I$ is the edge table index of the first and the last items of an incoming Doll. $H$ is the next vertex on the collision list. The vertices are the given indexes ordered incrementally by

**Table 3** Vertex hash table of $\mathcal{G}_5$

| Hash index | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Vertex table index | 2 | 0 | 1 | 4 | / |

**Table 4** Vertex table of $\mathcal{G}_5$

| **Index** | **0** | | **1** | | **2** | | **3** | | **4** | |
|---|---|---|---|---|---|---|---|---|---|---|
| Vertex ID | $v_1$ | | $v_2$ | | $v_3$ | | $v_4$ | | $v_5$ | |
| Out/In $w$ | 2 | 0 | 2 | 1 | 1 | 1 | 0 | 2 | 0 | 1 |
| O | 0 | 2 | 1 | 4 | 3 | 3 | / | / | / | / |
| I | / | / | 0 | 0 | 1 | 1 | 2 | 3 | 4 | 4 |
| H | 3 | | / | | / | | / | | / | |

$\phi_V = 5$

first arriving time. $\phi_V = 5$ means vertex table is full. If more vertices arrive, a new vertex table will be created, which begins with index 5 as the extension of existing vertex table.

**Edge hash table** Edge hash table: Dolha creates $m_e(m_e \geq |E|)$ size vertex hash table and uses function $H(*)$ map the outgoing vertex ID $u$ plus the incoming vertex ID $v$ of edge $\overrightarrow{uv}$ to the edge hash table index $H(\overrightarrow{uv})$. Same as the vertex hash table, Dolha stores the edge table index of the first edge on a collision list.

In Table 5, we have the same method as vertex hash table to deal with hash collision. $\overrightarrow{v_1v_4}$ has the same hash value 4 as $\overrightarrow{v_2v_5}$. In cell 4, we can find $\overrightarrow{v_1v_4}$'s edge table index 2 then find $\overrightarrow{v_2v_5}$'s edge table index.

**Edge table *E*** Dolha creates $m_e(m_e \geq |E|)$ size vertex table and uses a empty cell flag denoted by $\phi_E$. Initially, $\phi_E = 0$. We denote the vertex table index for new coming edge $\overrightarrow{uv}$ as $E(\overrightarrow{uv})$. Let $E(\overrightarrow{uv}) = \phi_E$ and increase $\phi_E$ by 1. In each edge table cell, Dolha stores the vertex table indexes $V(u)$ and $V(v)$, the weight $w(\overrightarrow{uv})$, the time stamp $t(\overrightarrow{uv})$, the previous and next edge table index for outgoing Doll, the previous and next edge table index for incoming Doll and the edge table index of the next edge on collision list.

Table 6 shows the edge table of $\mathcal{G}_5$ in Figure 6. $w$ is the weight and $t$ is the time stamp. Vertex index indicates the outgoing and incoming vertices of the edge. $O$ is the edge table index for the next and the previous items of outgoing Doll and $I$ is the edge table index of next and previous items of incoming Doll. $H$ is the next edge on the collision list.

### 4.2 Dolha snapshot construction

When an edge $(\overrightarrow{uv}; t; w)$ comes:

- Map the edge $\overrightarrow{uv}$ into edge hash table cell $H(\overrightarrow{uv})$.
- If $H(\overrightarrow{uv})$ is empty, $\overrightarrow{uv}$ does not exist in $D_s$. If $H(\overrightarrow{uv})$ is not empty, traverse the collision list of cell $H(\overrightarrow{uv})$ in edge hash table. If $\overrightarrow{uv}$ is found, $\overrightarrow{uv}$ exists; if not, $\overrightarrow{uv}$ does not exist.

There are two possible operations:
**If $\overrightarrow{uv}$ does not exist in $D_s$**

- Add $\overrightarrow{uv}$ into edge table cell $E(\overrightarrow{uv})$ and the collision list of $H(\overrightarrow{uv})$.
- Map the vertices $u$, $v$ into vertex hash table $H(u)$, $H(v)$.
- If $H(u)$ is empty, add ID $u$ into vertex table cell $V(u)$. If $H(u)$ is not empty, traverse the collision list of cell $H(u)$ in vertex hash table. If find match ID, then we update vertex table $V(u)$ of $u$; if not, add $u$ into vertex table cell $V(u)$ and collision list of $H(u)$.
- Do the same operation for $v$.

**Table 5** Edge hash table of $\mathcal{G}_5$

| Hash index | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Edge table index | 1 | 0 | 3 | / | 2 | / |

– Add $\overrightarrow{uv}$ into the end of outgoing Doll of $u$ and incoming Doll of $v$.

**If $\overrightarrow{uv}$ exists in $D_s$:**

– Set $t(\overrightarrow{uv}) = t$ and $w(\overrightarrow{uv}) = w(\overrightarrow{uv}) + w$.
– Delete $\overrightarrow{uv}$ from outgoing Doll of $u$ and incoming Doll of $v$
– If $\overrightarrow{uv}$ has positive weight after this update:
– Add $\overrightarrow{uv}$ into the end of outgoing and incoming Dolls.
– if $\overrightarrow{uv}$ has zero or negative weight after this update:
– Delete $\overrightarrow{uv}$ from edge table.
– If there is not any item in both Doll of $u$ or $v$, delete $u$ or $v$.

For example, at time 6, edge $\overrightarrow{v_3v_5}$ is received. We use $H(v_3v_5) = 3$ to get the edge hash table index and then to find edge $\overrightarrow{v_3v_5}$ as a new edge. We write the empty cell index 5 of edge table into hash table and check the two vertices by using vertex hash table. We locate the $V(2)$ for $v_3$ and $V(4)$ for $v_5$ on vertex table and get the last item of outgoing Doll $E(3)$ together with the last item of incoming Doll $E(4)$. We update both the last items of outgoing and incoming Doll to 5 then move to the edge table. We update the next item of outgoing Doll to 5 in $E(3)$ and update the next item of incoming Doll to 5 in $E(4)$. Finally, we write $w$, $t$, (2, 4), (3, /) and (4, /) into $E(5)$.

At time 7, edge $\overrightarrow{v_1v_2}$ comes. It is already on the edge table. We first update the $w$ and $t$ at $E(0)$ and remove $\overrightarrow{v_1v_2}$ from both of the Dolls then add it to the end of Dolls.

At time 8, edge $\overrightarrow{v_1v_4}$ carries negative weight and $w$ is 0 after the update. We remove $E(2)$ from the outgoing and incoming doll and update the related indexes, then we empty the cell 2 of edge table and put the index 2 into the empty edge cell list. At time 9, edge $\overrightarrow{v_1v_2}$ is deleted and $v_1$ has neither out nor in edges. We empty cell 0 of vertex table and put the index 0 into the empty vertex cell list.

## 4.3 Time and space cost

### 4.3.1 Time cost

Algorithm 1 shows how Dolha process one incoming edge.

From line 3 to 14, we maintain the edge hash table to check the existence of incoming edge $\overrightarrow{uv}$. According to [29], if we hash n items into a hash table of size n, the expected maximum list length is $O(\log n/\log\log n)$. In the experiment, more than 99% collision list is less than $\log n/\log\log n$, more than 90% collision list is shorter than 5. Hash table achieves amortized $O(1)$ time cost for 1 item insertion, deletion and update which is much faster than sorted table. Here the time cost is $O(1)$.

**Table 6** Edge table of $\mathcal{G}_5$

| **Index** | **0** | | **1** | | **2** | | **3** | | **4** | | **5** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $w$ | 1 | | 1 | | 1 | | 1 | | 1 | | / | |
| $t$ | 1 | | 2 | | 3 | | 4 | | 5 | | / | |
| Vertex index | 0 | 1 | 1 | 2 | 0 | 3 | 2 | 3 | 1 | 4 | / | / |
| O | / | 2 | / | 4 | 0 | / | / | / | 1 | / | / | / |
| I | / | / | / | / | / | 3 | 2 | / | / | / | / | / |
| H | / | | / | | 4 | | / | | / | | / | |

$\phi_E = 5$

**Algorithm 1** Dolha snapshot edge processing.

**Input**: Streaming graph $\mathcal{G}$
**Output**: Dolha snapshot structure of $\mathcal{G}$

1 **for** *each incoming edge* $(\overrightarrow{uv}; t; w)$ *of* $\mathcal{G}$ **do**
2     **Check existence of $\overrightarrow{uv}$:**
3     Map $\overrightarrow{uv}$ into $H(\overrightarrow{uv})$.
4     **if** $H(\overrightarrow{uv})$ *is null* **then**
5         $\overrightarrow{uv}$ does not exist
6     **else**
7         Traverse the collision list from $E^H_A(\overrightarrow{uv})$.
8         **if** *reach null and no match for* $\overrightarrow{uv}$ **then**
9             $\overrightarrow{uv}$ does not exist
10         **else**
11             $\overrightarrow{uv}$ exists
12     **if** $\overrightarrow{uv}$ *does not exist* **then**
13         **Update collision list of $\overrightarrow{uv}$:**
14         **if** $H(\overrightarrow{uv})$ *is empty* **then**
15             Let $E^H_A(\overrightarrow{uv}) = E(\overrightarrow{uv})$
16         **else**
17             Let $E^H_N(\overrightarrow{uv}^-) = E(\overrightarrow{uv})$
18         **Check existence of $u$:**
19         Map the vertices $u$ into $H(u)$
20         **if** $H(u)$ *is null* **then**
21             Add $u$ into vertex table $V(u)$ and let $V^H_A(u) = V(u)$
22         **else**
23             Traverse the collision list from $E^H_A(u)$.
24             **if** *reach null and no match for* $u$ **then**
25                 Add $u$ into vertex table $V(u)$ and let $V^H_N(u^-) = V(u)$
26         **Do the same operation for $v$ same as $u$**
27         Add $\overrightarrow{uv}$ into edge table $E(\overrightarrow{uv})$
28         **Add $\overrightarrow{uv}$ into outgoing Doll:**
29         **if** *both* $E^O_A(u)$ *and* $E^O_\Omega(u)$ *are null* **then**
30             Let $E^O_A(u) = E(\overrightarrow{uv})$ and $E^O_\Omega(u) = E(\overrightarrow{uv})$
31         **if** *neither* $E^O_A(u)$ *nor* $E^O_\Omega(u)$ *is null* **then**
32             Let $E^O(\overrightarrow{uv}^-) = E^O_\Omega(u)$ and $E^O_N(\overrightarrow{uv}^-) = E(\overrightarrow{uv})$ and $E^O_P(\overrightarrow{uv}) = E^O(\overrightarrow{uv}^-)$ and $E^O_\Omega(u) = E(\overrightarrow{uv})$
33         **Add $\overrightarrow{uv}$ into incoming Doll same as outgoing Doll**
34     **if** $\overrightarrow{uv}$ *exists* **then**
35         Let $w(\overrightarrow{uv}) += w$ and $t(\overrightarrow{uv}) = t$
36         **Delete $\overrightarrow{uv}$ from outgoing Doll:**
37         **if** $\overrightarrow{uv}$ *is the first item of outgoing Doll* **then**
38             Let $E^O_A(\overrightarrow{uv}) = E^O_N(\overrightarrow{uv})$ and $E^O_P(\overrightarrow{uv}^+) = null$
39         **if** $\overrightarrow{uv}$ *is the last item of outgoing Doll* **then**
40             Let $E^O_\Omega(\overrightarrow{uv}) = E^O_P(\overrightarrow{uv})$ and $E^O_N(\overrightarrow{uv}^-) = null$
41         **else**
42             Let $E^O_N(\overrightarrow{uv}^-) = E^O_N(\overrightarrow{uv})$ and $E^O_P(\overrightarrow{uv}^+) = E^O_P(\overrightarrow{uv})$
43         **Delete $\overrightarrow{uv}$ from incoming Doll same as outgoing Doll**
44         **if** $w(\overrightarrow{uv}) > 0$ **then**
45             Add $E(\overrightarrow{uv})$ into the end of outgoing Doll and incoming Doll
46         **else**
47             **Delete $\overrightarrow{uv}$**
48             Delete $E(\overrightarrow{uv})$ and flag $E(\overrightarrow{uv})$ as empty cell
49             **if** *there is no item on outgoing Doll or incoming Doll of* $u$ **then**
50                 Delete $V(u)$ and flag $V(u)$ as empty cell
51             **if** *there is no item on outgoing Doll or incoming Doll of* $v$ **then**
52                 Delete $V(v)$ and flag $V(v)$ as empty cell

If $\overrightarrow{uv}$ is a new edge, from line 16 to 22, the vertex hash table is maintained to check the existence of two vertices $u$ and $v$. In this step, we do two hash table look up and it costs $O(1)$ time. From line 23 to 29, we write $\overrightarrow{uv}$ into edge table then add it into the end of outgoing and incoming Dolls. The time complexity here is the same as the insertion on double linked list which is also $O(1)$.

If $\overrightarrow{uv}$ exists, from line 31 to 38, we will first update the weight and time stamp of $\overrightarrow{uv}$, and then delete it from outgoing and incoming Dolls. This step costs the same time as deletion on double linked list which is also $O(1)$. From line 39 to 40, if updated weight is positive, we add the $\overrightarrow{uv}$ to the end of both two Dolls which costs $O(1)$. If the updated weight is zero or negative, we delete $\overrightarrow{uv}$ completely then delete $u$ and $v$ if they have 0 in and out degrees. Line 41 to 46 shows the deletions and this step also costs $O(1)$.

Overall, for each incoming edge processing, the time complexity of Dolha is $O(1)$.

### 4.3.2 Space cost

Dolha snapshot structure needs one $|V|$ cells vertex hash table, one $|V|$ cells vertex table, one $|E|$ cells edge hash table and one $|E|$ cells edge table. Dolha also needs a $\log |V|$ bits integer for one vertex index and $\log |E|$ bits for one edge index.

**Vertex hash table:** Each cell only stores one vertex index. The space cost here is $\log |V| \times |V|$.

**Edge hash table:** Each cell only stores one edge index. The space cost here is $\log |E| \times |E|$.

**Vertex table:** Each cell stores vertex ID, in and out weights one $\log |V|$ bits vertex index for collision list, four $\log |E|$ bits edge indexes for Dolls. The space cost here is $(\log |V| + 4 \times \log |E|) \times |V|$.

**Edge table:** Each cell stores weight, time stamp, one $\log |E|$ bits edge index for collision list, two $\log |V|$ bits vertex index for in and out vertices, four $\log |E|$ bits edge indexes for Dolls. The space cost here is $(2 \times \log |V| + 5 \times \log |E|) \times |E|$ .

In total, Dolha needs $(2\times\log |V|+4\times\log |E|)\times|V|+(2\times\log |V|+5\times\log |E|)\times|E|$ bits for the data structure. Since usually $|V| \ll |E|$, the space cost of Dolha snapshot structure is $O(|E| \log |E|)$.

## 5 Dolha persistent structure

### 5.1 Dolha persistent data structure

Using Dolha, we could construct a persistent structure $D_p$ based on $D_s$. $D_p$ contains all the snapshot's information of the snapshots of $\mathcal{G}$. It has the same structure as $D_s$ except for the time travel list.

**Definition 8** (**Time Travel List**) An edge $\overrightarrow{uv}$ may appear in streaming graph $S$ multiple times with different time stamp. Time travel list $T$ is a single linked list that links all the edges $\overrightarrow{uv}$ which share same outgoing and incoming vertices. In $T$, each edge has an index points to its previous appearance in the stream.

$D_p$ also has four index-value tables: the vertex hash table, vertex table and edge hash table (same as in $D_s$). In each cell of the edge table, $D_p$ has a extra value to indicate the previous item on the time travel list.

### 5.2 Dolha persistent construction

#### 5.2.1 Incoming edge processing

When an edge $\sigma(\overrightarrow{uv}; t; w)$ comes:

- Check the existence of $\overrightarrow{uv}$, the same as in Dolha snapshot.

**If $\overrightarrow{uv}$ does not exist in $D_p$:**

- The operation is exactly the same as in Dolha snapshot.

**If $\overrightarrow{uv}$ exists in $D_p$:**

- Use edge hash table to find the existing edge table index $E(\sigma')$ of $\overrightarrow{uv}$.
- Insert edge $\sigma$ as a new edge into edge table, and set the time travel list index as $E(\sigma')$.
- Update the edge table index of $\overrightarrow{uv}$ on the edge hash collision list.

**Algorithm 2** Dolha persistent edge processing.

**Input**: Streaming graph $\mathcal{G}$
**Output**: Dolha persistent structure of $\mathcal{G}$

```
1  for each incoming edge σ(uv→; t; w) of G do
2      Check existence of uv→:
3      if uv→ does not exist then
4          Insert uv→
5      if uv→ exists in cell E(σ') then
6          Insert E(σ) as new edge and let w(σ) = w(σ) + w(σ')
7          Let E_P^T(σ) = E(σ')
8      if value of H(uv→) in edge hash table is null then
9          Let E_A^H(uv→) = E(σ)
10     else
11         Let E_N^H(uv→⁻) = E(σ)
```

Tables 7 and 8 show the Dolha persistent's edge hash table and edge table of $\mathcal{G}$ in Window 0. The vertex hash table and vertex table of Dolha persistent are similar like Dolha snapshot and so is the new edge coming. But for the updating process of edge $\overrightarrow{v_1 v_2}$ at time 6, we add the update as new edge into $E(6)$ and then update the edge hash table to latest. By using the time travel list, all the updates on $\overrightarrow{v_1 v_2}$ are linked.

#### 5.2.2 Sliding window update

When the window slides the $i$th step, we have the start-time $t_s = t_0 + (i-2) \times \theta$ and the end-time $t_e = t_0 + (i-1) \times \theta$ of expired edges which need to be deleted from the edge table. Since the edge table is naturally ordered by time, we can find the last expired edge denoted as $E(\sigma_e)$ at $t_e$ in $O(\log S)$ time. By using the edge hash table, we can find the latest update

**Table 7** Edge hash table of window 0

| Hash index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Edge table Index | 1 | / | 3 | / | 5 | 4 | / | 2 | 6 | / |

**Table 8** Edge table of window 0

| Index | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $w$ | 1 | | 1 | | 1 | | 1 | | 1 | | 1 | | 2 | | / | | / | | / | |
| $t$ | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | / | | / | | / | |
| V | 0 | 1 | 1 | 2 | 0 | 3 | 2 | 3 | 1 | 4 | 2 | 4 | 0 | 1 | / | / | / | / | / | / |
| O | / | 2 | / | 4 | 0 | 6 | / | 5 | 1 | / | 3 | / | 2 | 7 | / | / | / | / | / | / |
| I | / | / | / | 6 | / | 3 | 2 | 7 | / | 5 | 4 | / | 1 | 8 | / | / | / | / | / | / |
| H | / | | / | | / | | / | | / | | / | | / | | / | | / | | / | |
| T | / | | / | | / | | / | | / | | / | | 0 | | / | | / | | / | |

of $E(\sigma_\Omega)$ and traversal back by the time travel list. For each $E(\sigma_n)(e < n \leq \Omega)$ on time travel list, let $w_n = w_n - w_e$. If each $w_n \leq 0$, then delete all the $E(\sigma_n)$. Then delete each $E(\sigma_m)(0 < m \leq e)$ on time travel list. The same operations are adopted for all the edges from $t_e$ to $t_s$. For every deleted edge, if it is the first or last item of Doll, its associated cell in vertex table should be updated and its index set to *null*. If all its Doll indexes are *null* in the vertex cell, the vertex will be deleted and the cell will be flagged as empty.

As shown in Figure 7, when the window slides from 0 to 1, it means the edges before $t_4$ will be expired. First, we can binary search the edge table to locate the first unexpired edge index 3 since the table is sorted by time stamp. Then, we start to delete the expired edges from cell 3. We use the hash table to check if there is any unexpired update for the expired edges. For example, $\overrightarrow{v_1v_2}$ has unexpired update at time 7, so we minus the expired weight from cell 6.

Table 9 shows the edge table of Dolha persistent at Window 1. The first 3 expired edges have been deleted. At time 8, $\overrightarrow{v_1v_4}$ with negative weight arrives, but there is no positive $\overrightarrow{v_1v_4}$ in this window. In this case, $\overrightarrow{v_1v_4}$ will not be saved. At time 9 and 10, $\overrightarrow{v_1v_2}$ has either negative or zero weights, and $\overrightarrow{v_1v_4}$ has positive weight at time 7, thus we keep the record and link them by the time travel linked list.

**Space recycle** Due to the chronological ordered edge table, the expired edges are always sorted continuously and before the sections of the unexpired edges. We could always recycle the space occupied by the expired edges; we won't need infinite space to save the continuous streaming but only need the maximum number of edges in each window. For instance, in Table 9, we can re-use the cell from 0 to 1 for next window update and there will be enough space as long as there are no more than 9 edges in 1 window.

### 5.3 Time and space cost

The time cost of Dolha persistent is the sum of the time cost of hash table, Doll and time travel list. For each incoming edge, the hash table cost and Doll cost are $O(1)$ as discussed in Dolha snapshot and the time travel list cost is also $O(1)$ same as insertion on single linked list. Overall, the time cost for one edge processing is $O(1)$.

To store all the information of streaming $S$, Dolha persistent structure needs one $|V|$ cells vertex hash table, one $|V|$ cells vertex table, one $|S|$ cells edge hash table and one $|S|$ cells edge table. In total, Dolha needs $(2 \times \log|V| + 4 \times \log|S|) \times |V| + (2 \times \log|V| + 5 \times \log|S|) \times |S|$ bits plus $\log|S| \times |S|$ for time travel list. The space cost of Dolha persistent structure is $O(|S| \log|S|)$.

Table 9 Edge table of window 1

<table>
<tr><th>Index</th><th colspan="2">0</th><th colspan="2">1</th><th colspan="2">2</th><th colspan="2">3</th><th colspan="2">4</th><th colspan="2">5</th><th colspan="2">6</th><th colspan="2">7</th><th colspan="2">8</th><th colspan="2">9</th></tr>
<tr><td>$w$</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">-1</td><td colspan="2">0</td><td colspan="2">/</td></tr>
<tr><td>$t$</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">4</td><td colspan="2">5</td><td colspan="2">6</td><td colspan="2">7</td><td colspan="2">9</td><td colspan="2">10</td><td colspan="2">/</td></tr>
<tr><td>V</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>2</td><td>3</td><td>1</td><td>4</td><td>2</td><td>4</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td><td>/</td><td>/</td></tr>
<tr><td>O</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>5</td><td>/</td><td>/</td><td>3</td><td>/</td><td>/</td><td>7</td><td>6</td><td>8</td><td>7</td><td>/</td><td>/</td><td>/</td></tr>
<tr><td>I</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>/</td><td>5</td><td>4</td><td>/</td><td>/</td><td>7</td><td>6</td><td>8</td><td>7</td><td>/</td><td>/</td><td>/</td></tr>
<tr><td>H</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td></tr>
<tr><td>T</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">/</td><td colspan="2">6</td><td colspan="2">7</td><td colspan="2">/</td></tr>
</table>

# 6 Algorithms on Dolha

In this section, we will discuss the application of graph algorithms on both Dolha snapshot structure and persistent structure.

## 6.1 Algorithms on Dolha snapshot

### 6.1.1 Query primitives

Dolha snapshot structure supports all the 4 graph query primitives.

**Edge query** For a given pair of vertices IDs $(u, v)$, to query the weight and time stamp of edge $(\overrightarrow{uv})$ is the same as the existence checking of $(\overrightarrow{uv})$ in the insertion. Using the edge hash table, we can find $E(\overrightarrow{uv})$ on edge table and return $w$ and $t$. As proved before, the time cost of hash table checking is amortized $O(1)$.

**Vertex query** Similar to the edge query, by using vertex hash table, we can locate a given vertex $u$ on the vertex table in $O(1)$ time and return the query result.

**1-hop successor query and 1-hop precursor query** For a given vertex ID $u$, Dolha first perform vertex query to find $V(u)$ in $O(1)$ time. Then we have the head edge index $E_A^O(u)$ from the outgoing Doll. From $E(\sigma) = E_A^O(u)$, we can use $E_N^O(\sigma)$ to acquire all edges on the outgoing Doll iteratively, and add the incoming vertex indexes of these edges into set $\{V(v)\}$. The IDs of $\{V(v)\}$ can be found in the vertex table and then returned as the results of 1-hop successor query. The 1-hop precursor query is similar as successor query only using the incoming Doll instead. The time cost of Doll iteration depends on the outgoing or incoming degree $d$ of the given $u$. The total time cost of both the 1-hop successor query and the 1-hop precursor query is $O(d)$.

**Chronological doll** In Dolha structure, the Doll is maintained in chronological order. The result list of 1-hop successor query or 1-hop precursor query is sorted by the time stamps. The chronological Doll reduces the search space in some time related queries. For example, in Figure 4, we have a candidate edge $(\overrightarrow{uv}; t)$ that matches $(\overrightarrow{dc}; \epsilon_4)$ and look for the candidate edges of $(\overrightarrow{ce}; \epsilon_5)$. Since the timing order constrain $\epsilon_5 \prec \epsilon_4$, we first check the time stamp of first edge on $v$'s outgoing Doll in $O(1)$ time. If the time stamp is equal to or larger than $t$, it means there is no match for $(\overrightarrow{ce}; \epsilon_5)$. If the time stamp is less than $t$, we can search

from the first edge on $v$'s outgoing doll until the time stamp is equal to or larger the time stamp than $t$.

### 6.1.2 Directed triangle finding

With the 4 graph query primitives, most graph algorithms run on Dolha. The 1-hop successor query and 1-hop precursor query associated with edge query support all the BFS or DFS based algorithms like reachability query, tree parsing, shortest path query, subgraph matching and triangle finding. Here, the application on the triangle finding will be elaborated as a case of a common graph query on streaming graph.

To query the directed triangle on Dolha, we can use the edge iterator method. During the Dolha snapshot construction, we can add one out degree counter and one in degree counter for each vertex. For each edge ($\overrightarrow{uv}$) incoming edge, the minimal candidate set $\{j\}$ is constructed between $v$'s successor set and $u$'s precursor set. Then check each $j$ in set $\{j\}$ that if there is a ($\overrightarrow{ju}$) or ($\overrightarrow{vj}$) existing in the edge table by edge query. The set containing all existing ($\overrightarrow{uv}$, $\overrightarrow{vj}$, $\overrightarrow{ju}$) is the query result.According to [28], the time complexity of triangle finding on whole graph is $O(\sum_{\overrightarrow{uv}\in E}\min\{d_{in}(u), d_{out}(v)\})$, so the time cost is $O(\min\{d_{in}(u), d_{out}(v)\})$ for updating each edge.

**Algorithm 3** Continuous directed triangle finding on Dolha snapshot.

**Input**: Dolha snapshot structure of $\mathcal{G}$ with out and in degree counter
**Input**: Streaming Graph $\mathcal{G}$
**Output**: Directed triangles in $\mathcal{G}$

```
1 for each new coming edge uv of G do
2     if in degree of u ≤ out degree of v then
3         for each vertex j in u's precursor set do
4             if vj exsits in edge table then
5                 Put (uv, ju, vj) into result set
6     else
7         for each vertex j in v's successor set do
8             if ju exsits in edge table then
9                 Put (uv, ju, vj) into result set
```

## 6.2 Algorithms on Dolha persistent

### 6.2.1 Query primitives

Dolha persistent structure also supports all the 4 graph query primitives both on the latest snapshot and persistent perspective of $\mathcal{G}$:

**Edge query** For a given pair of vertices IDs $(u, v)$, the latest update for the edge $\overrightarrow{uv}$ could be found by using edge hash table. Once the latest update of edge $\overrightarrow{uv}$ is found, we could use time travel list to retrieve all the updates of $\overrightarrow{uv}$ in current window.

**Vertex query** The vertex query on Dolha persistent is exactly the same as snapshot structure.

**1-hop successor query and 1-hop precursor query:** For a given vertex ID $u$, the outgoing or incoming Doll of $u$ may contain duplicates of edges. To query the successor of $u$ on Dolha persistent, it's better to start from the last item of outgoing Doll $E^O_\Omega(u)$ which is definitely the latest outgoing edge from $u$. Let $E(\overrightarrow{uv}) = E^O_\Omega(u)$, then we add $v$ to the result set and use the time travel link of $\overrightarrow{uv}$ to flag all the previous update records of $\overrightarrow{uv}$. Then we traversal the outgoing doll and do the same operation for each unflagged edge as $\overrightarrow{uv}$. 1-hop precursor query is the same as successor query, the only different is to use the incoming Doll instead. Both of the two lists are sorted by time naturally.

### 6.2.2 Time related queries

**Time constrained pattern query** Given time period $(t, t')$, the essential part of time constrained pattern query is to find the all the edges with time stamp $(t \le t_{\overrightarrow{uv}} \le t')$ on snapshot $G'_t$. The chronological edge table allows us to locate the first edge $E(\sigma_t)$ at time $t$ and the last edge $E(\sigma'_t)$ at time $t'$ in $O(\log S)$ time. Then we can run Algorithm 4 to construct the adjacency list of the candidate subgraph of time constrained pattern query. Also we could construct a Dolha snapshot structure to store the candidate subgraph by using Algorithm 5. The time cost of candidate subgraph construction is $O(\log S + S')$ and the space cost is $O(S')$ ($S'$ is the incoming edge number of $(t, t')$). We can run any isomorphism algorithm on the candidate subgraph structure to get the final query result.

**Algorithm 4** Adjacency list construction for candidate subgraph of time constrained pattern query.

```
    Input: edges between σ_t and σ'_t in edge table
    Input: Dolha persistent structure of 𝒢
    Output: Adjacency list of candidate subgraph
 1  for each edge σ(uv→; t; w) from E(σ'_t) to E(σ_t) do
 2  |  if flag! = 3 then
 3  |  |  for each edge on the time travel list after σ do
 4  |  |  |  Let flag = 3
 5  |  |  if flag == 2 or flag == 0 then
 6  |  |  |  Put u into candidate vertex set
 7  |  |  |  for each edge σ_O on outgoing Doll of u do
 8  |  |  |  |  if flag! = 3 then
 9  |  |  |  |  |  for each edge on the time travel list after σ_O do
10  |  |  |  |  |  |  Let flag = 3
11  |  |  |  |  |  Let flag+ = 1
12  |  |  |  |  |  Put the incoming vertex of σ_O into the outgoing neighbor list of u
13  |  |  if flag == 1 or flag == 0 then
14  |  |  |  Put v into candidate vertex set
15  |  |  |  for each edge σ_I on incoming Doll of v do
16  |  |  |  |  if flag! = 3 then
17  |  |  |  |  |  for each edge on the time travel list after σ_I do
18  |  |  |  |  |  |  Let flag = 3
19  |  |  |  |  |  Let flag+ = 2
20  |  |  |  |  |  Put the outgoing vertex of σ_I into the incoming neighbor list of v
```

**Algorithm 5** Dolha snapshot construction for candidate subgraph of time constrained pattern query.

**Input**: edges between $\sigma_t$ and $\sigma_t'$ in edge table
**Input**: Dolha persistent structure of $\mathcal{G}$
**Output**: Dolha snapshot of candidate subgraph

1 Construct a Dolha snapshot structure $D_t'$ with vertex and edge size $|E(\sigma_t') - E(\sigma_t)|$ **for** *each edge* $\sigma(\overrightarrow{uv}; t; w)$ *from* $E(\sigma_t')$ *to* $E(\sigma_t)$ **do**
2   **if** $flag! = 1$ **then**
3     **for** *each edge on the time travel list after* $\sigma$ **do**
4       Let $flag = 1$
5     Insert $\sigma$ into $D_t'$

**Structure constrained time query** Given a sequence of directed edges $Q\{q_1, q_2, ..., q_m\}$, for each edge $q_n$ in $Q$, we can use the edge hash table to locate the latest update $E(q_n)$ in $\mathcal{G}$ and use time travel list to find the time period set $T_n$ when edge $q_n$ appears. Then we join all the time period sets to find the result time period set. The Algorithm 6 shows that the time complexity is $O(m \times p \times \log(m \times p))$ ($p$ is the average amount of the occurrence of a certain edge in $S$).

**Algorithm 6** Structure constrained time query.

**Input**: a sequence of directed query edges $Q\{q_1, q_2, ..., q_m\}$
**Input**: Dolha persistent structure of $\mathcal{G}$
**Output**: Time period set $T$ that match the query structure

1 Let $t_e = null$ and $t_s = null$
2 Let chronological order set $T_c = \phi$
3 **for** *each edge* $q_n$ *in* $Q$ **do**
4   Use edge hash table to find the latest update $E(q_n)$
5   **for** *each edge* $q_n^t$ *on time travel list of* $q_n$ *from* $E(q_n)$ **do**
6     **if** $w_n^t > 0$ **then**
7       Let $t_s = t_n^t$
8       **if** $t_e == null$ **then**
9         Let $t_e = t_n^t$
10     **if** $w_n^t \leq 0$ **then**
11       **if** $t_s! = null$ **then**
12         Put $t_s$ flag as $s$ and $t_e$ flag as $e$ into $T_c$
13         Let $t_e = null$ and $t_s = null$
14 **for** *each item* $c_e$ *that flagged as e in* $T_c$ **do**
15   **if** *there are m continuous s items on the left of c* **then**
16     Let $c_s$ = the closest left $s$ item
17     Put $(c_s, c_e)$ into $T$

# 7 Experimental evaluation

## 7.1 Experiment setup

We evaluate Dolha snapshot and Dolha persistent structure separately.

In Dolha snapshot experiment, we compare Dolha snapshot with adjacency matrix in hash table and adjacency list in hash table. Since TCM is based on adjacency matrix in hash table and the java project GraphStream is based on adjacency list in hash table, we believe the comparison to these two general GraphStream structures could reflect the performance of Dolha properly. And we also use the dynamic graph structure PMA[1] [2]for comparation. For the four structures, we first compare the average operation time cost and space cost and then compare the speed of query primitives.

We use the same hash function (MurmurHash) for all the structures and build the same vertex hash table and vertex table for all four structures so they all share the same vertex operation time cost and accuracy. Because the full adjacency matrix is too large, we compress the matrix in certain ratios that costs similar space as Dolha. That makes TCM become an approximation structure and we take account of the relative error.

In Dolha persistent experiment, since there is no similar system for comparison, we build an adjacency list in hash table with an extra time line which stores all the edge update information. We use the adjacency list as baseline method to compare with Dolha persistent on the speed of sliding window update, query primitives and time related queries.

#### 7.1.1 Dataset

1. **DBLP** [10]**:** DBLP dataset contains 1,482,029 unique authors and 10,615,809 time-stamped coauthorship edges between authors (about 6 million unique edges). It's a directed graph and we assign each streaming edge with weight 1.
2. **GTGraph** [15]**:** We use the graph generator toll GTGraph to generate a directed graph. We use the R-MAT model generate a large network with power-law degree distributions, add weight 1 to for each edge and use the system clock to get the time-stamp. The generated graph contains 30 million vertices and 1 billion streaming edges.
3. **Twitter** [6]**:** We use the Twitter link structure data as a directed streaming graph. It has 56 million vertices and 2 billion edges with weight 1 assigned to each edge.
4. **CAIDA** [5]**:** CAIDA Internet Anonymized Traces 2015 Data-set obtained from www.caida.org. The network data contains 445,440,480 records of communication as edges (about 100 million unique edges) concerning 2,601,005 different IP addresses as vertices.

We use 4 datasets above for the Dolha snapshot experiment: The DBLP, GTGraph and Twitter are used for Dolha snapshot experiments and DBLP and CAIDA are used for Dolha persistent experiments.

#### 7.1.2 Environment

All experiments are performed on a server with dual 8-core CPUs (Intel Xeon CPU E5-2640 v3 @ 2.60GHz) and 128 GB DRAM memory, running CentOS. All the data structures are implemented in C++.

### 7.2 Dolha snapshot experimental results

#### 7.2.1 Construction

Firstly, we compare the average time and space cost of records stream graphs on four structures:TCM, GraphStream and Dolha. In real world scenario, the insertion, deletion and

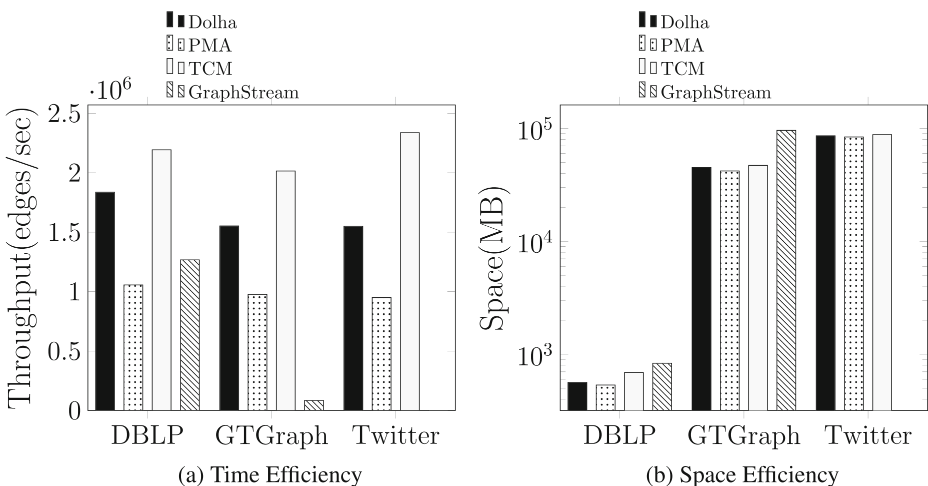
update operations are usually coming randomly and the average stream processing speed is the key performance indicator of the system and all three operations time costs on Dolha are $O(1)$. We load the datasets 2 times as insertion and update and set the weight to $-3$ for last loading as deletion. Then we calculate the average time of datasets loading as the stream processing time cost and present it in the form of operations per second. During the data loading, we record the actual memory consumption when the edges are fully loaded. The results are shown in Figure 12.

In the DBLP dataset, Dolha processing speed reaches 1,837,357 operations per second which almost same as TCM ( 2,192,715 operations per second) and faster than GraphStream (1,266,815 operations per second) and PMA (1,055,950 operations per second). Due to the preset compress ratio, the memory cost of TCM is 690MB which is similar to Dolha's 563MB. The GraphStream costs 833MB which is worse than Dolha.

In the GTGraph dataset, the performance remains the same. The TCM is the fastest structure with 2,014,768 operations per second and Dolha is not far behind with 1,552,536 operations per second. The speed of GraphStream drops significantly to 85,441 operations per second and the space cost reaches 96GB which is way higher than Dolha's 45GB and TCM's 47GB. The PMA structure has the slower speed 976,950 but better memory consumption.

In the Twitter dataset, the GraphStream runs out memory since the enormous space cost for the maintenance of sorted list. The performances of Dolha and TCM are steady. Dolha costs 86GB memory and reaches 1,550,197 operations per second while the TCM costs 88GB and reaches 2,336,785 operations per second.The PMA structure has 950,197 operations per second speed and $84GB$ memory cost.

The time costs show that Dolha is slightly slower on stream processing speed than the TCM but significantly faster than the GraphStream and PMA. The slight latency to TCM is acceptable, since the TCM is an approximation structure and Dolha is an exact structure. The space costs show that Dolha could process 2 billion streaming edges in less than 90GB memory.




(a) Time Efficiency (b) Space Efficiency

**Figure 12** Time and space cost for the 3 streaming graph structure

### 7.2.2 Query primitives

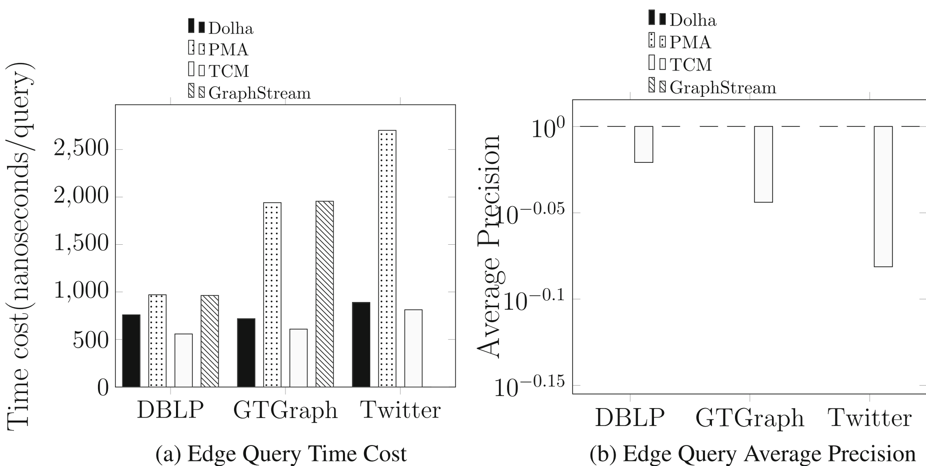
In this part, we will compare the query primitives speed on the four systems: the vertex query, the edge query, 1-hop successor query and 1-hop precursor query. The time-related query and sliding window update are not supported by the other two structures and the time costs are depended on the given parameters, so we have not run experiment on these two queries.

**Vertex query** The four structures share the same vertex hash table and vertex table, so the vertex query speeds are same. We run 25 random vertex queries and the time cost is 14,146 nanoseconds in total. It means the average vertex query is 566 nanoseconds per query.

**Edge query** We run 50 random edge queries with four structures on each dataset. The results show that speed of edge query on Dolha is similar as on TCM with 0 relative error and much faster than on GraphStream and PMA.

**1-hop successor query and 1-hop precursor query** We randomly choose 25 vertices and run 1-hop successor query and 1-hop precursor query with four structures on each dataset. Since the speed depends on the size of results set, the average query speed is calculated as nanoseconds per result. The TCM has almost 0 average precision on these queries and the slowest query speed. Among the four structures, Dolha has the best performance with fast query speed and 100% precision.

Compare to the GraphStream and PMA, Dolha has great advantages on the average stream processing time cost, edge query speed, 1-hop successor query and 1-hop precursor query speed. Dolha is slightly slower than the TCM with similar space cost on average stream processing time cost, space cost, edge query speed but faster on 1-hop successor query and 1-hop precursor query. On the other hand, the Dolha is an exact structure and the TCM is an approximation structure Figures 13, 14 and 15.




(a) Edge Query Time Cost

(b) Edge Query Average Precision

**Figure 13** Time cost and average precision rate for edge queries

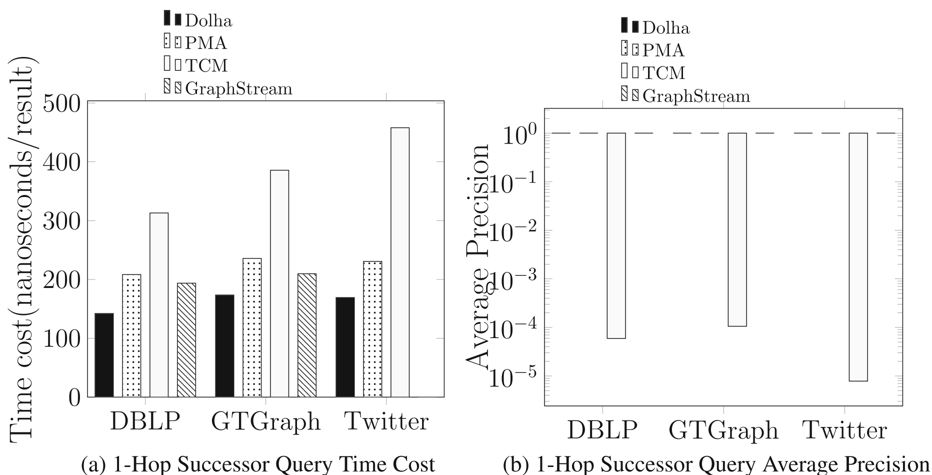



**Figure 14** Time cost and average precision for 1-hop successor query

### 7.2.3 Graph algorithms

**BFS** We run BFS for 10 random vertices on DBLP and GTGraph datasets. Since the precision of TCM is below the acceptable range, we only compare between Dolha and GraphStream. The experiment results shows that Dolha is about 30% faster than GTGraph. In DBLP dataset, the average BFS time cost of Dolha is 419 milliseconds, the time cost of GraphStream is 599 milliseconds and the time cost of PMA is 580 milliseconds. In GTGraph dataset, the average BFS time cost of Dolha is 41,432 milliseconds, the time cost of GraphStream is 67,049 milliseconds and the time cost of PMA is 67,149 milliseconds. In Twitter dataset, the average BFS time cost of Dolha is 58,816 milliseconds and the time cost of PMA is 687,618 milliseconds Figure 16

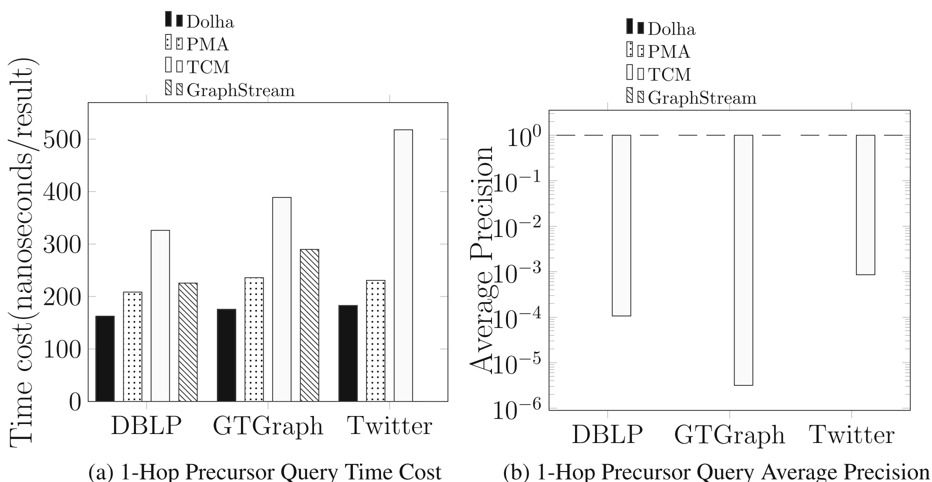
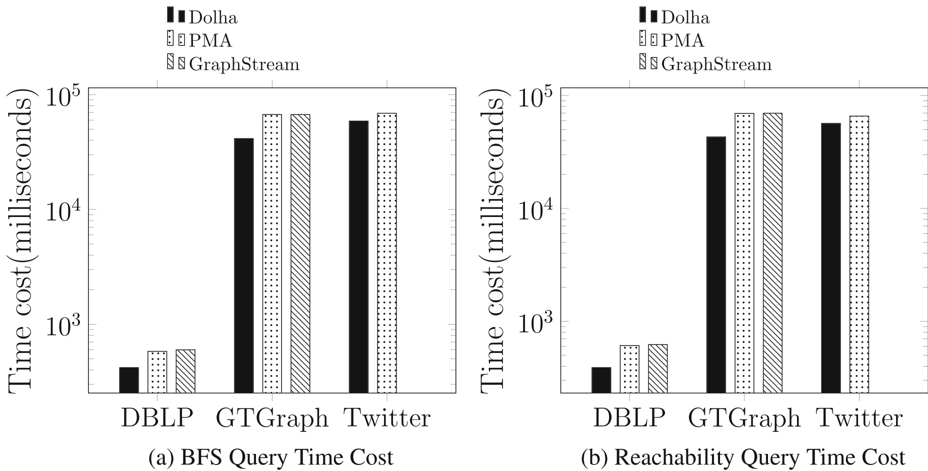



**Figure 15** Time cost and average precision for 1-hop precursor query



(a) BFS Query Time Cost

(b) Reachability Query Time Cost

**Figure 16** Time cost for BFS and reachability query

**Reachability query** We run reachability query for 10 random pairs of vertices on DBLP and GTGraph datasets. Since TCM is a compressed adjacency matrix, it has a high false positive rate for reachability queries. TCM answers most reachability queries as "reachable" and the precision only depends on the reachable rate of given queries. In this case, we only compare between Dolha and GraphStream. The experiment results shows that Dolha is about 40% faster than GTGraph. In DBLP dataset, the average reachability queries time cost of Dolha is 389 milliseconds, the time cost of GraphStream is 622 milliseconds and the time cost of PMA is 610 milliseconds. In GTGraph dataset, the average BFS time cost of Dolha is 43,019 milliseconds, the time cost of GraphStream is 69,703 milliseconds and the time cost of PMA is 69,524 milliseconds. In Twitter dataset, the average BFS time cost of Dolha is 56,732 milliseconds and the time cost of PMA is 64,265 milliseconds Figure 16

**Directed triangle finding** We run continuous the directed triangle finding algorithm on DBLP and GTGraph 1 billion date set using Dolha snapshot and GraphStream. For DBLP dataset, Dolha processes 759,866 edge updates per-second and GraphStream only processes 238,095 edge updates per-second. For GTGraph 1 billion date set, Dolha is capable to deal 129,853 throughput edges per-second but GraphStream only deals less than 10,000 throughput edges per-second.

## 7.3 Dolha persistent experimental results

### 7.3.1 Construction and sliding window update

We set window length $= \frac{1}{10}|S|$, slide length $= \frac{1}{5}$ window length as W1 and slide length $= \frac{1}{50}$ window length as W2. Then we load the DBLP and CAIDA dataset with / without sliding window update. Figure 17 shows the through-puts of Dolha persistent and adjacency list plus time-line with / without sliding window update.

On DBLP date set, Dolha persistent reaches 2,008,420 edges update per second without sliding window update, 1,979,889 edges update per second in W1 and 1,961,238 edges update per second in W2. The adjacency list plus time-line only can process 1,120,269 edges

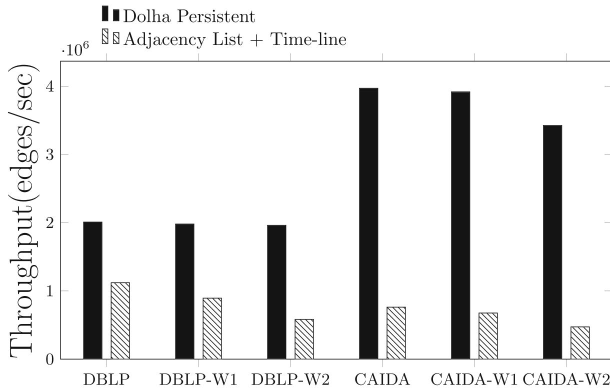



**Figure 17** Edge throughput without and with time window update

update per second without sliding window update, 893,795 edges update per second in W1 and 583,367 edges update per second in W2. Comparing with no sliding window update, the processing speed of W1 drops 1% and the processing speed of W2 drops 2.3%.

On CAIDA dataset, Dolha persistent reaches 3,969,514 edges update per second without sliding window update, 3,917,037 edges update per second in W1 and 3,425,009 edges update per second in W2. The results are way better than the adjacency list plus time-line's speeds: 761,834 edges update per second without sliding window update, 676,077 edges update per second in W1 and 472,953 edges update per second in W2. Comparing with no sliding window update, the processing speed of W1 drops 1% and the processing speed of W2 drops 1.3%.

The outstanding high speed is due to the high duplicated edge rate on CAIDA dataset. We set the edge hash table as the same size as edge table, but the unique edge number is only $\frac{1}{4}$ of total stream edge number. This reduces the hash collision significantly. In this way, we do not need to check the vertices by using vertex hash table when processing the duplicated edge update

#### 7.3.2 Query primitives

The query primitives of DBLP on Dolha persistent are exact the same as on Dolha snapshot. Here we only compare the CAIDA with adjacency list plus time-line Figure 18.

**Vertex query** The two structures use the same vertex hash table and vertex table. We run 25 random vertex queries and the average vertex query is 605 nanoseconds per query.

**Edge query** We run 50 random edge queries on both data structures. The result shows that Dolha persistent is 5 times faster than adjacency list plus time-line.

**1-hop successor query and 1-hop precursor query** We randomly choose 25 vertices and run 1-hop successor query and 1-hop precursor query on the two structures. Dolha persistent is slighly faster than adjacency list plus time-line.

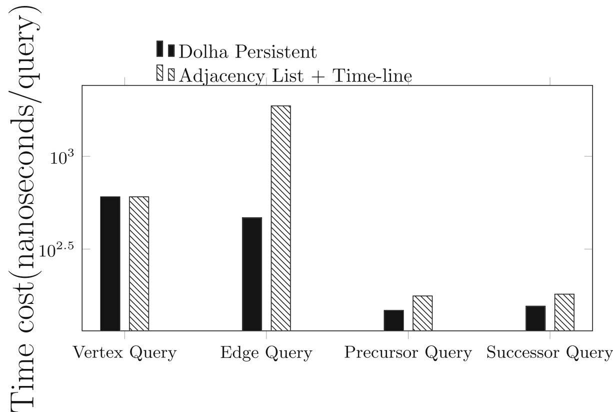
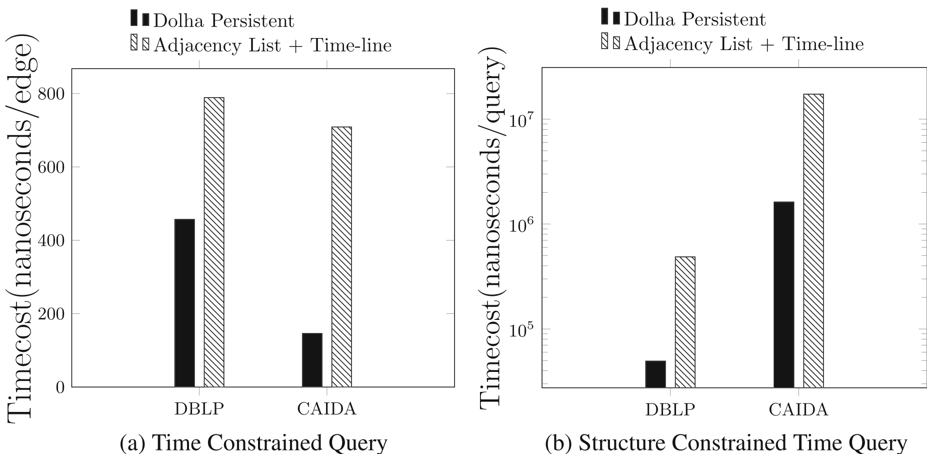



**Figure 18** Query primitives on CAIDA

### 7.3.3 Time related queries

**Time constrained pattern query** For time constrained pattern query, we randomly choose 3 pairs of time-stamps as the time constraints and extract the eligible edges to form a candidate subgraph. Figure 19a shows the average speeds of forming the list of candidate subgraphs with Dolha persistent and adjacency list plus time-line. In DBLP, we reach 457 nanoseconds per edge to extract the candidate subgraph into a Dolha snapshot, meanwhile the adjacency list plus time-line can only construct 789 nanoseconds per edge into an adjacency list. In CAIDA, the speed reaches 146 nanoseconds per edge while the adjacency list plus time-line can only process 709 nanoseconds per edge.

**Structure constrained time query** To compare structure constrained time query, we randomly choose 5 query edge sets, each of which has 5 edges. The average query time of




(a) Time Constrained Query (b) Structure Constrained Time Query

**Figure 19** Time related query

Dolha persistent is 49,378 nanoseconds per query on DBLP and 1,623,200 nanoseconds per query on CAIDA. The average query time of adjacency list plus time-line is 486,576 nanoseconds per query on DBLP and 17,312,871 nanoseconds per query on CAIDA Figure 19b.

## 8 Conclusions and future of work

We have proposed an exact streaming graph structure Dolha which could maintain high speed and high volume streaming graph in linear time cost and near-linear space cost. We have shown that Dolha is a structure suitable for general proposes, and supports the query primitives in the major graph algorithms. Dolha persistent structure, as a variant of Dolha, supports the sliding window update and time-related queries efficiently. The experiment results have proved that Dolha has better performance than the other streaming graph structures.

In the future, we plan to extend Dolha into two fields. One extention is using Dolha to process the streaming RDF graph data and queries. The other extention is implanting Dolha into GPU for parallel processing.

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